Innovation, Capital Structure, and the Boundaries of the Firm[*]

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Abstract

We study how interactions between financing and investment decisions can shape firm boundaries in innovative markets. In particular, we model innovative projects as growth options and ask whether they are best operated inside large incumbent firms (Integration) or in separate, specialized firms (Non-Integration). Starting from a standard theoretical framework, in which value-maximizing corporate investment and financing decisions are jointly determined, we show that Integration best protects assets in place value, while Non-Integration best protects the value of the innovation option and maximizes financial flexibility. These forces drive different organizational equilibria depending on firm and product market characteristics. We also show that alliances organized as licensing agreements or revenue sharing contracts sometimes better balance the different sources of value, and thus may dominate more traditional forms of organization.

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1 Introduction

Financial economists have long understood that the presence of pre-existing debt financing can cause managers to make suboptimal investment decisions when they act in the interests of existing shareholders (Fama and Miller (1972), Myers (1977)).[1] At the same time, a vast literature in industrial organization and the economics of organizations has explored how firm boundaries are important for determining which types of firms conduct innovative activities (see, e.g., Aghion and Tirole (1994), Anton and Yao (1995), Mathews and Robinson (2008), Robinson (2008), Inderst and Mueller (2009), and Fulghieri and Sevilir (2010)).[2] The goal of this paper is to study how debt financing and investment distortions interact with the dynamics of innovation to determine the boundaries of the firm.

We study a model in which a set of assets in place and a related innovative project modeled as an investment option must be optimally organized and financed. Both corporate activities are subject to cash flow risk represented by a standard diffusion process. We find closed-form valuation equations under two competing organizational forms, reflecting the fact that innovation can be organized inside an existing firm holding the assets in place (innovation by a large or "integrated" firm) or can be organized as a separate firm controlling the option but without assets in place (innovation by a small "non-integrated" firm). The debt associated with financing assets in place delays exercise of the option due to "debt overhang," which naturally pushes the innovative activity out of the large firm in order to minimize investment distortions and maximize capital structure flexibility. Several realistic features of product markets work against this tendency, however, and the optimal organization of innovative activities is determined in equilibrium by balancing the investment and financing benefits of non-integration against factors that favor developing the innovative project inside the large firm.

The first such factor is cannibalization. Because the innovative activity is related to the assets in place, exercising the innovation option naturally involves cannibalizing some of the profits associated with the assets in place. In general, the large, integrated firm will internalize the cannibalization costs in its decision to exercise the option, but the small, stand-alone firm will not. Thus, there can be important product market externalities associated with having a non-integrated firm undertake the innovative activity. As cannibalization increases, this tends to push the optimal organizational design towards innovation in the large firm.

The second factor is obsolescence risk. This embodies the idea that the innovative activity can

[1] More recent contributions that explore these effects quantitatively include Mello and Parsons (1992), Mauer and Triantis (1994), Hennessy (2004), and Hackbarth (2009). All of these papers study departures from the Modigliani and Miller (1958) framework with no taxes, bankruptcy costs, informational asymmetries, or agency costs.

[2] This line of research follows in the footsteps of earlier work on the boundaries of the firm in general, such as Coase (1937), Williamson (1973, 1979), Grossman and Hart (1986), and Hart and Moore (1990).

potentially be preempted by competing firms. Thus, the option can jump to being worthless if a competitor innovates first. This alternative type of innovation, e.g., by a competitive fringe, can also adversely affect the value of assets in place.

While the cannibalization effect is straightforward, the effect of obsolescence risk and the aforementioned cash flow risk on the optimal organizational design is not. An increase in cash flow risk tends to increase the critical level of cannibalization needed to make integration optimal. This is in line with existing conventional wisdom that smaller firms tend to be more innovative in riskier markets. However, an increase in obsolescence risk tends to decrease the critical level of cannibalization, making integration more likely. This difference is driven by the fact that the two types of risk have opposite effects on the optimal exercise time for the option. Cash flow risk increases option value and makes it optimal to delay exercise, while obsolescence risk speeds up exercise to avoid preemption. Because the value of assets in place is increasing and concave in exercise time, it turns out that protecting assets in place value by choosing integration is more important when exercise occurs earlier, i.e., when cash flow risk is low or obsolescence risk is high.

These results imply starkly different empirical implications for the optimal location of innovation depending on the type of risk. Greater cash flow risk, such as uncertainty about market size, should tend to predict innovation by independent firms, while a greater risk of preemption due to innovation by other firms should predict innovation by larger, more established firms.

We also explore how optimal organizational design responds, for example, to the corporate tax rate, the magnitude of bankruptcy costs, and the relative size of the growth option. Our analysis shows that non-integration is more likely to be optimal the greater is the corporate tax rate or the smaller is the level of bankruptcy costs. As the tax rate is higher or bankruptcy costs are lower, the importance of debt overhang and financial flexibility are magnified. This leads to non-integration since capital structure decisions can then be made independently. We also show that non-integration is more likely to be optimal the larger is the relative magnitude of the growth option. When the growth option is larger relative to assets in place, the negative effects of debt overhang and financial inflexibility are magnified, which leads to non-integration in order to preserve option value. These results should prove useful for future empirical investigations of the organization of innovation.

Finally, we investigate how hybrid organizational forms, such as alliances, could fit into our framework. In particular, starting from our non-integrated case, we investigate the effect of a financial alliance that takes the form of a licensing or revenue sharing contract. This is modeled as a proportion of the cash flows from the innovative project following exercise that is promised to the large firm. Because the small firm still bears the full cost of exercise, this has the effect of causing the small firm to exercise the option later, closer to the time that is optimal to protect the value of assets in place. We show that since the firms choose the licensing fraction ex ante to maximize their joint expected surplus, the added flexibility of this contract can be quite valuable, such that the alliance form often dominates both non-integration and integration. Intuitively, separating the two firms removes debt overhang and increases financial flexibility, while the licensing contract ameliorates the resulting problem of sub-optimal joint profit maximization in the exercise decision.

Although we are the first to provide an analysis of how capital structure and organizational design jointly affect the dynamics of innovation, our work is closely related to a number of papers, including Grenadier and Weiss (1997), Berk, Green, and Naik (2004), Carlson, Fisher, and Giammarino (2006), and others that model innovative activities as real options. Common features of such models include investment irreversibility, stochastic cash flows related to underlying market/economic uncertainty, and, more recently, competitive implications. However, these models generally consider the operation of such projects in isolation (i.e., without consideration for optimal organization) and without debt financing, whereas we focus on the joint value effect of an integration decision for an innovative project together with value relevant capital structure decisions.

This paper also relates to several other contributions to the capital structure literature. Mello and Parsons (1992) are the first to examine the interactions of investment and financing decisions in a real options model. They show that capital structure can have a significant impact on operating decisions. Tserlukevich (2008) and Titman and Tsyplakov (2010) build dynamic models in which firms can issue debt to exercise a sequence of growth options. Leland (1998) studies the joint determination of capital structure and asset risk, while Chen and Manso (2010) emphasize that incorporating macroeconomic risk can increase agency costs of debt substantially. Hackbarth and Mauer (2010) study the relation between the priority structure of corporate debt and firms' investment and financing decisions. Morellec and Schuerhoff (2010) focus on the implications of asymmetric information on the financing and timing of corporate investment.

Perhaps most closely related in spirit is a set of papers on capital structure and project finance in static settings without innovation options or product market interactions. For example, John (1986), John (1993), and Flannery, Houston, and Venkataraman (1993) consider how to optimally organize and finance two projects with varying payoff correlations or risk structures in the presence of agency-induced and tax-based incentives. More recently, Leland (2007) analyzes the role of net tax benefits for spin-offs and mergers in a model with correlated cash flows but without agency problems. Finally, Shah and Thakor (1987) study the optimality of project finance when there is asymmetric information about project quality, while Chemmanur and John (1996) show that separate incorporation or project finance can be used to optimally allocate control rights.[3]

The remainder of the paper is organized as follows. Section 2 presents the model. Section 3 solves the model by deriving the optimal policies and resulting values for the Non-Integrated form (i.e., separated, small firm, and separated, large firm) and the Integrated form (i.e., the large firm with the innovation option). Section 4 provides implications and results of the model. Section 5 provides extensions to consider alliances and alternative assumptions about capital structure flexibility. Section 6 concludes, and technical details can be found in the Appendix.

2 The Model

We consider an economy with two types of corporate activities: a set of "assets in place" operated by an existing firm, and an "innovation option" whose potential future cash flows are subject to the same underlying economic uncertainty. At every point in time t, assets in place generate uncertain cash flows, X_t, which evolve over time according to a geometric Brownian motion with drift μ and volatility σ. Agents are risk neutral and discount cash flows at a constant, risk-free rate r with $\mu < r$.

To irreversibly exercise the innovation (or growth) option, its owner (either the existing firm or a new firm established to operate the option) has to spend investment costs $\kappa > 0$, after which time it receives from the new assets an incremental stream of cash flows equal to πX_t, where $\pi > 0$. However, once the option is exercised, the existing firm's future cash flows from assets in place are decreased by some fraction γ, which represents a cannibalization effect of the growth option on the existing business. This leaves the assets in place generating $(1+\gamma) X_t < X_t$ in cash flows thereafter.

Furthermore, there is a risk of preemption, e.g., because innovation by another firm or firms makes the product underlying the option's cash flows obsolete. Specifically, the innovation opportunity may randomly "die" during any time interval dt with a constant probability $\rho\, dt$. A firm facing no rival for a growth opportunity can optimally time investment without consideration of outside factors. On the other hand, a firm whose expansion opportunity is at risk of preemption may invest earlier in order to itself preempt challengers. In addition, we assume that when the option is lost due to obsolescence, the existing firm's assets in place suffer a cannibalization effect δ, such that its cash flows thereafter are equal to $(1 + \delta) X_t < X_t$. This represents in reduced form the competitive effect of innovation by rival firm(s) that triggered the option's obsolescence.

At the beginning of the game, an organizational design choice is made. We consider two possible designs. In the Integrated design, the existing firm owns both the assets in place and the growth option (hereafter "I" or the "integrated" firm). Thus, it chooses the time at which to exercise the

[3] Like us, Habib and Mella-Barral (2010) study organizational design in a dynamic setting but focus on information transmission through mergers and alliances without considering either capital structure effects or innovation options.

option taking into account its effect on assets in place. In the Non-Integrated design, the existing firm (hereafter "L" or the "large" firm in the Non-Integrated case) continues to own the assets in place, but ownership of the growth option is placed with a new, completely separate firm. The new firm (hereafter "S" or the "small" firm) thus chooses the time of exercise independently.

We assume corporate taxes are paid at a rate τ on operating cash flows less interest, and full offsets of corporate losses are allowed. Thus, capital structure can affect firm value. We further assume that in the case of bankruptcy (which is triggered by an endogenous default decision on behalf of equityholders), future cash flows are reduced by a proportion α of the base cash flows of the defaulting firm, where the base cash flows do not include any cannibalization effects. Thus, if the existing firm defaults at any point, future cash flows from assets in place are reduced by $\alpha\, X_t$ (regardless of whether the option has been exercised or preempted). Similarly, if the firm holding the growth option defaults after exercise, future cash flows from the new assets are reduced by $\alpha\, \pi\, X_t$.

In both the Integrated and Non-Integrated case, the existing firm makes a once and for all capital structure choice immediately after the organizational design has been chosen. In particular, it chooses an instantaneous, perpetual coupon payment C so as to maximize its total firm value (which is equivalent to assuming it is all-equity financed ex ante and chooses a debt issuance that maximizes equity value). In the Non-Integrated case, the new firm chooses its capital structure at the time of option exercise; prior to exercise it has no cash flows and therefore is all-equity financed by assumption. After debt has been issued, equityholders can choose to default at any time.

Since debt is issued prior to exercise in the Integrated case, we assume the exercise time is chosen by the firm's equityholders to maximize their own value. In the Non-Integrated case, there is no debt prior to exercise so the chosen exercise time also maximizes equity value by definition.

3 Solution

To compare the two different organizational design regimes, we first solve in this section for contingent claim values. In a second step, we derive optimal financing and investment decisions in the cases of Non-Integration and Integration.

3.1 Non-Integration: The Small Firm

In the Non-Integrated case, the small firm generates no cash flows and makes no debt payments prior to investment. That is, the small firm is all-equity financed until exercise. Upon exercise, the small firm's assets in place start generating a perpetual stream of after-tax cash flows $(1-\tau)\, \pi\, X_t$

at each time t. If no debt is issued, the small firm's unlevered value after exercise is given by:

$$\mathrm{E}\left[\int_t^{\infty} e^{-r\,(s-t)}\,(1-\tau)\,\pi\,X_s\,ds\right] \;=\; \pi\,\Lambda\,X_t, \tag{1}$$

where $\mathrm{E}[\cdot]$ is the expectation operator, and $\Lambda = (1-\tau)/(r-\mu)$ is the after-tax, growth-adjusted discount factor, which is similar to Gordon's growth formula with μ being the growth rate.

Since debt and equity are issued to finance the capital expenditure κ, the small firm's levered total value after exercise reflects the present value of the cash flows accruing until the default time, i.e., the after-tax cash flows $(1-\tau)\,\pi\,X_t$ plus the tax savings $\tau\,C_S^+$ (where C_S^+ is the coupon chosen by the firm at the time of exercise), and the present value of the cash flows accruing after default, i.e., $(1-\alpha)\,(1-\tau)\,X_t$. The small firm's equity value after exercise reflects the present value of the cash flows accruing until the default time, i.e., the after-tax cash flows $(1-\tau)\,(\pi\,X_t - C_S^+)$, and the present value of the cash flows accruing after default, i.e., 0 assuming strict adherence to absolute priority.

We denote equity and firm values when the small firm has exercised its option, issued debt with coupon payment C_S^+, and selected the default threshold $\underline{X}_S^+$, by E_S^+ and V_S^+ (note that, from here forward, use of $^+$ superscript denotes values relevant after option exercise). Based on standard arguments (see the Appendix), we can solve for the small firm's optimal decisions and its contingent claim values in closed-form, which are summarized in the following lemma.

Lemma 1 *Given the current value of cash flow X, the small firm's total firm value after investment equals for all $X \geq \underline{X}_S^+$:*

$$V_S^+(X) \;=\; \pi\,\Lambda\,X \;+\; \frac{\tau\,C_S^+}{r}\left(1-\left(\frac{X}{\underline{X}_S^+}\right)^{\vartheta'}\right) \;-\; \alpha\,\pi\,\Lambda\,\underline{X}_S^+\left(\frac{X}{\underline{X}_S^+}\right)^{\vartheta'}\;, \tag{2}$$

and its equity value after investment is for all $X \geq \underline{X}_S^+$ given by:

$$E_S^+(X) \;=\; \left(\pi\,\Lambda\,X - \frac{(1-\tau)\,C_S^+}{r}\right) \;-\; \left(\pi\,\Lambda\,\underline{X}_S^+ - \frac{(1-\tau)\,C_S^+}{r}\right)\left(\frac{X}{\underline{X}_S^+}\right)^{\vartheta'}\;, \tag{3}$$

where ϑ' is the negative characteristic root of the quadratic equation: $\frac{1}{2}\,x\,(x-1)\,\sigma^2 \,+\, x\,\mu \;=\; r$,

$$\vartheta' \;=\; \left(\tfrac{1}{2}-\mu/\sigma^2\right) \;-\; \sqrt{\left(\tfrac{1}{2}-\mu/\sigma^2\right)^2 + 2\,r/\sigma^2}\;. \tag{4}$$

The equity value-maximizing default threshold is:

$$\underline{X}_S^+ \;=\; \frac{\vartheta'}{\vartheta'-1}\,\frac{r-\mu}{r}\,\frac{C_S^+}{\pi}\;, \tag{5}$$

and the firm value-maximizing coupon payment equals:

$$C_S^+ \;=\; \pi\,X\,\frac{\vartheta'}{\vartheta'-1}\,\frac{r}{r-\mu}\left[1-\vartheta'\left(1-\alpha+\frac{\alpha}{\tau}\right)\right]^{1/\vartheta'}. \tag{6}$$

The first term in $V_S^+(X)$ is the value of assets in place in (1), the second term is the expected value of tax shield benefits from debt (which disappear if the firm defaults at $\underline{X}_S^+$), and the third term is the expected value of bankruptcy costs, which are triggered when the firm defaults at $\underline{X}_S^+$. To identify the sources of firm value, we will often refer jointly to the second and the third term as the firm's net tax benefits. For $E_S^+(X)$, the first term represents the expected value of after-tax cash flows to equityholders, while the second term subtracts the expected value of those cash flows conditional on default at $\underline{X}_S^+$, so that equityholders' claim value equals zero upon default.

Next, we define the stopping time $\mathcal{T}_Y > 0$ that determines the time at which obsolescence occurs. Let Y_t be the associated indicator function, which is equal to zero if $t < \mathcal{T}_Y$ and one otherwise. If $Y_t = 0$, an unexercised option may be exercised, but if $Y_t = 1$ the option is worthless. Working backwards, the value of the small firm prior to exercise, V_S, crucially depends on preemption risk or, more specifically, the distribution of $\mathcal{T}_Y$. As long as $Y_t = 0$, V_S equals the expected present value of the optimally levered firm value minus capital expenditure at the time of investment. We denote the investment threshold selected by shareholders by $\overline{X}_S$ and the first time for X to touch this threshold from below by $\mathcal{T}_G$. Thus, the small firm invests to maximize the value of its option:

$$V_S(X) \;=\; \sup_{\mathcal{T}_G} \mathrm{E}\left[\, 1_{\mathcal{T}_G < \mathcal{T}_Y}\, e^{-r\mathcal{T}_G} \left(V_S^+(X_{\mathcal{T}_G}) \;-\; \kappa\right)\right] \;, \tag{7}$$

where 1_ω represents the indicator function of the event ω. Because the firm does not produce any cash flows before investment, initial shareholders only receive capital gains of $\mathrm{E}\,[dV_S(X)]$ over each time interval dt prior to investment. The required rate of return for investing in the small firm is the risk-free rate r. Thus, the Bellman equation in the continuation region with $t < \mathcal{T}_Y$ is:

$$r\, V_S(X)\, dt \;=\; \mathrm{E}\,[dV_S(X)] \tag{8}$$

Applying Ito's lemma to expand the right-hand side of the Bellman equation, it is easy to show that the value of the small firm before investment and preemption satisfies:

$$r\, V_S(X) \;=\; \mu\, X\, \frac{\partial V_S(X)}{\partial X} \;+\; \frac{1}{2}\,\sigma^2 X^2\, \frac{\partial^2 V_S(X)}{\partial X^2} \;+\; \rho\,[0 - V_S(X)] \;, \tag{9}$$

where the last term reflects the impact of obsolescence risk on the growth option. In particular, the left-hand side of this equation reflects the required rate of return for holding the claim per unit of time. The right-hand side is the expected change in the claim value (i.e., the realized rate of return). These expressions are similar to those derived in standard contingent claims models. However, they contain the additional term, $\rho\,[0 - V_S(X)]$, which reflects the impact of losing the growth option. This term is the product of the instantaneous probability of obsolescence and the change in the value function occurring due to obsolescence.

The ordinary differential equation (9) is solved subject to the following boundary conditions. First, the value of equity at the time of investment is equal to the payoff from investment: $V_S(\overline{X}_S) = V_S^+(\overline{X}_S) - \kappa$. Second, as the level of the cash flow shocks tends to zero, the option to invest becomes worthless so that V_S satisfies: $\lim_{X \downarrow 0} V_S(X) = 0$. In addition, to ensure that investment occurs along the optimal path, the value of equity satisfies the optimality condition (smooth-pasting): $\partial V_S(\overline{X}_S)/\partial X\big|_{X=\overline{X}_S} = \partial V_S^+(\overline{X}_S)/\partial X\big|_{X=\overline{X}_S}$ at the endogenous investment threshold. Solving the small firm's problem yields the following results (see the Appendix):

Proposition 1 *Given the current value of cash flow X, the value of the non-integrated, small firm's optimal equity/firm value equals for all $X \leq \overline{X}_S$ and $t < \mathcal{T}_Y$:*

$$V_S(X) = \frac{\kappa}{\xi - 1}\left[\frac{\xi\, X}{\xi - 1}\,\frac{r-\mu}{1-\tau}\,\frac{\kappa}{\pi}\Big(1 + \frac{\tau}{1-\tau}\left(1 - \vartheta'\left(1 - \alpha + \alpha/\tau\right)\right)^{1/\vartheta'}\Big)\right]^{\xi}, \tag{10}$$

which can be re-written as:

$$V_S(X) = G_S(X) + NTB_S(X), \tag{11}$$

where $G_S(X)$ denotes value of the growth option for the small firm:

$$G_S(X) = \left(\pi\,\Lambda\,\overline{X}_S - \kappa\right)\left(\frac{X}{\overline{X}_S}\right)^{\xi}, \tag{12}$$

$NTB_S(X)$ denotes the value of net tax benefits for the small firm:

$$NTB_S(X) = \pi\,\Lambda\,\overline{X}_S\,\frac{\tau}{1-\tau}\left(1 - \vartheta'\left(1 - \alpha + \alpha/\tau\right)\right)^{1/\vartheta'}\left(\frac{X}{\overline{X}_S}\right)^{\xi}, \tag{13}$$

and $\overline{X}_S$ denotes the value-maximizing exercise threshold for the small firm:

$$\overline{X}_S = \frac{\xi}{\xi - 1}\,\frac{r-\mu}{1-\tau}\,\frac{\kappa}{\pi}\left(1 + \frac{\tau}{1-\tau}\left(1 - \vartheta'\left(1 - \alpha + \alpha/\tau\right)\right)^{1/\vartheta'}\right)^{-1}, \tag{14}$$

and where ξ is the positive characteristic root of the quadratic equation $\frac{1}{2}\,x\,(x-1)\,\sigma^2 + x\,\mu = r + \rho$,

$$\xi = \left(\tfrac{1}{2} - \mu/\sigma^2\right) + \sqrt{\left(\tfrac{1}{2} - \mu/\sigma^2\right)^2 + 2\,(r+\rho)/\sigma^2}\,. \tag{15}$$

Proposition 1 provides the value of of the small firm prior to exercise and preemption as the expected present value of the after-tax cash flows net of capital expenditures, G_S, and the net tax benefits, NTB_S, that are initiated at the time of investment $\mathcal{T}_G$. Several standard comparative statics results from the real options literature apply to V_S and $\overline{X}_S$. For example, the small firm's value is higher (lower), when the investment payoff π (investment cost κ) rises and hence its exercise threshold is lower (higher). Consistent with economic intuition, the proposition shows that V_S decreases with ρ because when preemption becomes more likely, all else equal, the small firm is less likely to invest. That is, a higher risk of preemption erodes the value of the small firm's option. As a result of the reduced option value, the small firm optimally exercises this option, in expectation, earlier when obsolescence risk is higher, i.e., $\partial \overline{X}_S / \partial \rho < 0$. Thereby, the small firm avoids being preempted in the states of the world in which it would otherwise continue to wait to invest. However, the small firm delays exercise when cash flow risk is higher, i.e., $\partial \overline{X}_S / \partial \sigma > 0$. Moreover, we see that $\partial \overline{X}_S / \partial \tau > 0$, even though higher corporate taxes provide more net tax benefits, because the first-order effect of higher taxes is substantially lower after-tax cash flows.[4]

3.2 Non-Integration: The Large Firm

Even though the large firm does not invest in the Non-Integrated case, its values and hence its value-maximizing decisions are more complex than those of the small firm. This is because the large firm is initially capitalized by both debt and equity and, more importantly, because it is not known at time zero whether the small firm will invest (i.e., $\mathcal{T}_G < \mathcal{T}_Y$) or will be preempted (i.e., $\mathcal{T}_G \geq \mathcal{T}_Y$). We denote the separated, large firm's equity and firm values when the small firm has exercised its option by E_L^+ and V_L^+ (again use of $^+$ superscript denotes values after option exercise). Correspondingly, let $\underline{X}_L^+$ denote the default threshold selected by shareholders. As in the previous section, we begin by deriving contingent claim values after exercise, which are gathered in the next lemma.

Lemma 2 *Given the large firm's initial coupon choice C_L and the current value of cash flow X, total firm value equals for all $t \geq \mathcal{T}_G$ and $X \geq \underline{X}_L^+$:*

$$V_L^+(X) \;=\; (1+\gamma)\,\Lambda\, X \;+\; \frac{\tau\, C_L}{r}\left(1 - \left(\frac{X}{\underline{X}_L^+}\right)^{\vartheta'}\right) \;-\; \alpha\, \Lambda\, \underline{X}_L^+ \left(\frac{X}{\underline{X}_L^+}\right)^{\vartheta'} \;, \tag{16}$$

and its equity value after investment is for all $X \geq \underline{X}_L^+$ given by:

$$E_L^+(X) \;=\; \left((1+\gamma)\,\Lambda\, X - \frac{(1-\tau)\, C_L}{r}\right) \;-\; \left((1+\gamma)\,\Lambda\, \underline{X}_L^+ - \frac{(1-\tau)\, C_L}{r}\right)\left(\frac{X}{\underline{X}_L^+}\right)^{\vartheta'} \;, \tag{17}$$

where ϑ' is the negative characteristic root of the quadratic equation: $\frac{1}{2}\, x\,(x-1)\,\sigma^2 \,+\, x\,\mu \;=\; r$,

$$\vartheta' \;=\; \left(\tfrac{1}{2} - \mu/\sigma^2\right) \;-\; \sqrt{\left(\tfrac{1}{2} - \mu/\sigma^2\right)^2 + 2\,r/\sigma^2} \;. \tag{18}$$

The equity value-maximizing default threshold is:

$$\underline{X}_L^+ \;=\; \frac{\vartheta'}{\vartheta' - 1}\,\frac{r-\mu}{r}\,\frac{C_L}{1+\gamma} \;. \tag{19}$$

[4] Notice also that $\partial \overline{X}_S / \partial \alpha > 0$ because with higher bankruptcy costs less debt will be optimally issued for a given cash flow level X and hence also at the optimal exercise threshold. Intuitively, more of the capital expenditures will be equity-financed and hence exercise optimally takes place, in expectation, later.

We denote the large firm's equity and firm values when the small firm has lost its option due to preemption, by E_L° and V_L° (note that from here forward, use of the $^\circ$ superscript denotes values relevant after the option has become obsolete due to preemption). Correspondingly, let $\underline{X}_L^\circ$ denote default threshold selected by shareholders. We can obtain the following analytic expressions.

Lemma 3 *Given the large firm's initial coupon choice C_L and the current value of cash flow X, total firm value equals for all $t \geq \mathcal{T}_Y$ and $X \geq \underline{X}_L^\circ$:*

$$V_L^\circ(X) \;=\; (1+\delta)\,\Lambda\,X \;+\; \frac{\tau\,C_L}{r}\left(1-\left(\frac{X}{\underline{X}_L^\circ}\right)^{\vartheta'}\right) \;-\; \alpha\,\Lambda\,\underline{X}_L^\circ\left(\frac{X}{\underline{X}_L^\circ}\right)^{\vartheta'} \quad , \tag{20}$$

and its equity value after investment is for all $X \geq \underline{X}_L^\circ$ given by:

$$E_L^\circ(X) \;=\; \left((1+\delta)\,\Lambda\,X-\frac{(1-\tau)\,C_L}{r}\right) \;-\; \left((1+\delta)\,\Lambda\,\underline{X}_L^\circ-\frac{(1-\tau)\,C_L}{r}\right)\left(\frac{X}{\underline{X}_L^\circ}\right)^{\vartheta'} \quad , \tag{21}$$

where ϑ' is the negative characteristic root of the quadratic equation: $\frac{1}{2}\,x\,(x-1)\,\sigma^2 \,+\, x\,\mu \;=\; r$,

$$\vartheta' \;=\; \left(\tfrac{1}{2}-\mu/\sigma^2\right) \;-\; \sqrt{\left(\tfrac{1}{2}-\mu/\sigma^2\right)^2+2\,r/\sigma^2} \;. \tag{22}$$

The equity value-maximizing default threshold is:

$$\underline{X}_L^\circ \;=\; \frac{\vartheta'}{\vartheta'-1}\,\frac{r-\mu}{r}\,\frac{C_L}{1+\delta} \;. \tag{23}$$

The results in Lemma 2 and Lemma 3 afford a similar interpretation as the ones in Lemma 1. For example, the main sources of firm value are again the value of assets in place and the value of net tax benefits. The main difference between them is the absence or presence of cannibalization effects. Notably, competitive cannibalization due to preemption ($\delta < 0$) or monopolistic cannibalization by the small firm ($\gamma < 0$) undermine the large firm's asset in place values and hence total firm value in (20).

Since equity value is the difference between firm and debt value, the cannibalization effects reduce equity value in (21) in lock step with (20). The effect of cannibalization on equity value can be seen most clearly in the equity value-maximizing default threshold. Thus, in contrast to (5) in Lemma 1, the default threshold after preemption in (23) has a different term, which adjusts equity's optimal default boundary by a multiplicative factor $1/(1+\delta) > 1$ instead of $1/\pi$.

Working backwards, the value of the large firm prior to exercise or obsolescence equals the expected present value of the levered firm values in three regions: (i) before investment and preemption, (ii) after investment, and (iii) after preemption. We denote the large firm's equity and firm values at time zero by E_L and V_L. Moreover, we denote the default threshold selected by shareholders in region (i) by $\underline{X}_L$ and the first time for X to touch this threshold from above by $\mathcal{T}_D$.

Because the large firm operates assets in place before the small firm's investment decision, its owners receive capital gains of $\mathrm{E}\,[dV_L(X)]$ and cash flows $(1-\tau)\,X + \tau\,C_L$ over each time interval dt. The required rate of return for investing in the large firm is the risk-free rate r. Thus, the Bellman equation in the continuation region with $t < \mathcal{T}_Y$ is:

$$r\,V_L(X)\,dt \;=\; \mathrm{E}\,[dV_L(X)] \,+\, [(1-\tau)\,X \,+\, \tau\,C_L]\,dt \tag{24}$$

Applying Ito's lemma to expand the right-hand side of the Bellman equation, it is immediate to derive that the value of the large firm before investment and preemption satisfies:

$$r\,V_L(X) \;=\; (1-\tau)\,X \,+\, \tau C_L \,+\, \mu\,X\,\frac{\partial V_L(X)}{\partial X} \,+\, \frac{1}{2}\,\sigma^2 X^2\,\frac{\partial^2 V_L(X)}{\partial X^2} \,+\, \rho\,[V_L^{\circ}(X) - V_L(X)]\;, \tag{25}$$

where the last term reflects the impact of the small firm's obsolescence risk on the large firm's value. The left-hand side of this equation reflects the required rate of return for holding the claim per unit of time. The right-hand side is the after-tax cash flow cum tax savings, $(1-\tau)\,X + \tau\,C_L$, plus the expected change in the claim value (i.e., the realized rate of return). These expressions are similar to those derived in standard contingent claims models. However, they contain the additional term, $\rho\,[V_L^{\circ}(X) - V_L(X)]$, which reflects the impact of preemption on the large firm's value (i.e., cannibalization of assets in place due to preemption and the resulting change in the default boundary). The new term on the right-hand side is the product of the instantaneous probability of obsolescence and the change in the large firm's value function at the time of obsolescence $\mathcal{T}_Y$.

The ordinary differential equation (25) is solved subject to the following boundary conditions. First, the value of the large firm at the time of the small firm's investment $\mathcal{T}_G$ is equal to the value of the large firm in Lemma 2 evaluated at the small firm's investment threshold (value-matching): $V_L(\overline{X}_S) = V_L^{+}(\overline{X}_S)$. Second, the value of the large firm at the time its shareholders default $\mathcal{T}_D$ is equal to its value of assets in place net of bankruptcy costs plus the expected effect on assets in place due to cannibalization by either the small firm or by the competitive fringe (value-matching): $V_L(\underline{X}_L) = (1-\alpha)\,\Lambda\,X_L + \rho\,\delta\,\Lambda\,[\underline{X}_L - \overline{X}_S(\underline{X}_L/\overline{X}_S)^{\xi}]/(r+\rho-\mu) + \gamma\,\Lambda\,\overline{X}_S(\underline{X}_L/\overline{X}_S)^{\xi}$.

Similar arguments lead to the large firm's equity value, $E_L(X)$. As we show in the Appendix, equity satisfies a similar differential equation as (25),[5] which has also a solution with unknown constants that are determined by the following boundary conditions. First, the value of the large firm's equity at the time of the small firm's investment $\mathcal{T}_G$ is equal to the value of the large firm's equity in Lemma 2 evaluated at the small firm's investment threshold (value-matching): $E_L(\overline{X}_S) = E_L^{+}(\overline{X}_S)$. Second, equity value at the time of default $\mathcal{T}_D$ is equal to zero under the absolute priority rule (value-matching): $E_L(\underline{X}_L) = 0$. In addition, to ensure that default occurs

[5] More specifically, *equity* cash flows $(1-\tau)\,(X - C_L)$ replace *firm* cash flows $(1-\tau)\,X \,+\, \tau C_L$ in equation (25).

along the optimal path, the value of equity satisfies the optimality (smooth-pasting) condition at the endogenous default threshold (see, e.g., Leland, 1994, 1998). Solving yields the following results.

Proposition 2 *Given the current value of cash flow X, the non-integrated, large firm's total value equals for all $X \in \left(\underline{X}_I, \overline{X}_S\right)$ and $t < \mathcal{T}_Y$:*

$$V_L(X) \;=\; AIP_L(X) \,+\, NTB_L(X)\;, \tag{26}$$

where the value of assets in place, AIP_L, is given by:

$$AIP_L(X) \;=\; \Lambda\, X \,+\, \frac{\rho}{r+\rho-\mu}\,\delta\,\Lambda\left[X - \left(\frac{X}{\overline{X}_S}\right)^{\xi}\overline{X}_S\right] + \gamma\,\Lambda\,\overline{X}_S\left(\frac{X}{\overline{X}_S}\right)^{\xi}, \tag{27}$$

and the value of net tax benefits, NTB_L, is given by:

$$\begin{aligned} NTB_L(X) \;=\; & \frac{\tau\, C_L}{r}\left(1 \,-\, \Delta(X) \,-\, \Sigma(X)\left(\frac{\overline{X}_S}{\underline{X}_L^{+}}\right)^{\vartheta'} - \frac{\rho}{r-\mu'}\,\Psi(X)\right) \\ & -\alpha\left(\Delta(X)\,\Lambda\,\underline{X}_L \,+\, \Sigma(X)\,\Lambda\,\underline{X}_L^{+}\left(\frac{\overline{X}_S}{\underline{X}_L^{+}}\right)^{\vartheta'} + \frac{\rho}{r-\mu'}\,\Lambda\,\underline{X}_L^{\circ}\,\Psi(X)\right)\;. \end{aligned} \tag{28}$$

The value of the non-integrated, large firm's equity equals for all $X \in \left(\underline{X}_L, \overline{X}_S\right)$ and $t < \mathcal{T}_Y$:

$$\begin{aligned} E_L(X) \;=\; & \left(\Lambda\, X \,-\, \frac{(1-\tau)\,C_L}{r}\right) + \frac{\rho}{r+\rho-\mu}\,\delta\,\Lambda\left[X - \Delta(X)\,\underline{X}_L - \Sigma(X)\,\overline{X}_S\right] \\ & +\gamma\,\Lambda\,\overline{X}_S\left(\frac{X}{\overline{X}_S}\right)^{\xi} - \frac{\rho}{r-\mu'}\left((1+\delta)\,\Lambda\,\underline{X}_L^{\circ} \,-\, \frac{(1-\tau)\,C_L}{r}\right)\Psi(X) \\ & -\Delta(X)\left(\Lambda\,\underline{X}_L \,+\, \gamma\,\Lambda\,\overline{X}_S\left(\frac{\underline{X}_L}{\overline{X}_S}\right)^{\xi} \,-\, \frac{(1-\tau)\,C_L}{r}\right) \\ & -\Sigma(X)\left((1+\gamma)\,\Lambda\,X \,-\, \frac{(1-\tau)\,C_L}{r}\right)\left(\frac{\overline{X}_S}{\underline{X}_L^{+}}\right)^{\vartheta'}\;, \end{aligned} \tag{29}$$

where $\overline{X}_S$ is the small firm's investment threshold in (14), the stochastic discount factors for default by the large firm and for investment by the small firm are given by:

$$\Delta(X) \;=\; \frac{X^{\xi}\,\overline{X}_S^{\vartheta} - X^{\vartheta}\,\overline{X}_S^{\xi}}{\underline{X}_L^{\xi}\,\overline{X}_S^{\vartheta} - \underline{X}_L^{\vartheta}\,\overline{X}_S^{\xi}}\;, \qquad \textit{and} \qquad \Sigma(X) \;=\; \frac{\underline{X}_L^{\xi}\,X^{\vartheta} - \underline{X}_L^{\vartheta}\,X^{\xi}}{\underline{X}_L^{\xi}\,\overline{X}_S^{\vartheta} - \underline{X}_L^{\vartheta}\,\overline{X}_S^{\xi}}\;, \tag{30}$$

the adjusted growth rate is $\mu' = \vartheta'\mu + \frac{1}{2}\,\vartheta'\,(\vartheta'-1)\,\sigma^2$, and the terms related to preemption risk are

$$\Psi(X) \;=\; \left(\frac{X}{\underline{X}_L^{\circ}}\right)^{\vartheta'} - \Delta(X)\left(\frac{\underline{X}_L}{\underline{X}_L^{\circ}}\right)^{\vartheta'} - \Sigma(X)\left(\frac{\overline{X}_S}{\underline{X}_L^{\circ}}\right)^{\vartheta'}\;, \tag{31}$$

and where ϑ' and ξ are given in (4) and (15), and ϑ is the negative characteristic root of the quadratic equation $\frac{1}{2}\,x\,(x-1)\,\sigma^2 \,+\, x\,\mu \;=\; r \,+\, \rho$,

$$\vartheta \;=\; \left(\tfrac{1}{2} - \mu/\sigma^2\right) \,-\, \sqrt{\left(\tfrac{1}{2} - \mu/\sigma^2\right)^2 + 2\,(r+\rho)/\sigma^2}\;. \tag{32}$$

Finally, the optimal (firm value-maximizing) coupon choice solves $\max_{C_L} V_L(X)$ *and the optimal (equity value-maximizing) default threshold solves* $\partial E_L(X)/\partial X|_{X=\underline{X}_L} = 0$.

Proposition 2 reports closed-form solutions for firm value and equity value when the large, non-integrated firm's cash flows from assets in place are affected by both diffusion risk (i.e., cash flow uncertainty) and jump risk (i.e., obsolescence/preemption uncertainty). The value of the large firm can again be broken down into two main parts, the value of assets in place, $AIP_L(X)$, and the value of net tax benefits, $NTB_L(X)$, which explains (26). Equations (27) and (28) provide the details for those two parts. The first term in (27) is the base cash flow value of assets in place, the second term represents the expected value of cannibalization due to preemption/obsolescence (which is offset by the possibility that the option will be exercised first, in which case obsolescence risk disappears as reflected in the term involving $\Sigma(X)$), and the third term represents the expected value of cannibalization due to the exercise of the growth option by the small firm.

In (28), the first line gives the expected value of tax shields, while the second gives the expected value of bankruptcy costs. The term in parentheses on the first line gathers terms involving the state prices for the various circumstances in which default can occur, which are: (a) when the firm reaches the boundary $\underline{X}_L$ before either obsolescence or option exercise occur (state price $\Delta(X)$, defined in (30)), (b) when the firm first exercises the option, then later defaults at $\underline{X}_L^+$ (state price involving $\Sigma(X)$, defined in (30), multiplied by the state price $(\overline{X}_S/\underline{X}_L^+)^{\vartheta'}$), and (c) when the option becomes obsolete prior to both exercise and default due to the instantaneous probability of obsolescence ρ, and the firm then later defaults at $\underline{X}_L^\circ$ (state price involving $\Psi(X)$ and $1/(r-\mu')$ is the appropriate discount factor for claim values that are contingent on $X^{\vartheta'}$ instead of X). Note that the term $\Psi(X)$, detailed in (42), corrects for the probability that the firm will default (the $\Delta(X)$ term) or that the option will be exercised (the $\Sigma(X)$ term) prior to obsolescence, at which point obsolescence risk disappears. The second line in (28) analogously accounts for the present value of bankruptcy costs for the various states in which default can occur (recall that bankruptcy costs are proportional to cash flows at the time of default, whereas tax shield cash flows are fixed from time zero).

The formula for equity value, (29), accounts for the value of the base cash flows to equity (the first term), the possible reductions due to cannibalization (the next two terms, involving δ and γ), and the possibility that all equity cash flows will cease under the various circumstances in which default can occur (the three terms involving $\Psi(X)$, $\Delta(X)$, and $\Sigma(X)$ in conjunction with $(\overline{X}_S/\underline{X}_L^+)^{\vartheta'}$). Finally, the result notes that the optimal values of the large firm's coupon choice C_L and its pre-investment/pre-obsolescence default threshold $\underline{X}_L$ do not have explicit analytical solutions. However, they can easily be computed by maximizing firm value at time zero with respect to the coupon, and by imposing the smooth-pasting condition for equity value at $\underline{X}_L$, which can be expressed analytically as a non-linear equation.

3.3 Integration: Large Firm's Value with Growth Option

In the Integrated case, we solve for a single firm value which combines the different projects of the two separated firms under one umbrella. Thus, the firm's value-maximizing decisions attempt to strike a balance of their effect on assets in place value, growth option value, and net tax benefits. As in the previous section, it is not known at time zero whether the integrated firm will invest (i.e., $\mathcal{T}_G < \mathcal{T}_Y$) or will be preempted (i.e., $\mathcal{T}_G \geq \mathcal{T}_Y$). We denote the integrated firm's equity and firm values after option exercise by E_I^+ and V_I^+. Correspondingly, let $\underline{X}_I^+$ denote default threshold selected by shareholders. The next lemma presents contingent claim values after exercise.

Lemma 4 *Given the integrated firm's initial coupon choice C_I and the current value of cash flow X, total firm value equals for all $t \geq \mathcal{T}_G$ and $X \geq \underline{X}_I^+$:*

$$V_I^+(X) \;=\; (1+\pi+\gamma)\,\Lambda\,X \;+\; \frac{\tau\,C_I}{r}\left(1-\left(\frac{X}{\underline{X}_I^+}\right)^{\vartheta'}\right) \;-\; \alpha\,(1+\pi)\,\Lambda\,\underline{X}_I^+\left(\frac{X}{\underline{X}_I^+}\right)^{\vartheta'}\;, \tag{33}$$

and its equity value after investment is for all $X \geq \underline{X}_I^+$ given by:

$$E_I^+(X) \;=\; \left((1+\pi+\gamma)\,\Lambda\,X-\frac{(1-\tau)\,C_I}{r}\right)-\left((1+\pi+\gamma)\,\Lambda\,\underline{X}_I^+-\frac{(1-\tau)\,C_I}{r}\right)\left(\frac{X}{\underline{X}_I^+}\right)^{\vartheta'}\;, \tag{34}$$

where ϑ' is given in (4) and the equity value-maximizing default threshold is:

$$\underline{X}_I^+ \;=\; \frac{\vartheta'}{\vartheta'-1}\,\frac{r-\mu}{r}\,\frac{C_I}{1+\pi+\gamma}\;. \tag{35}$$

Notice that, if the integrated firm is preempted, then it loses its option and hence its unlevered value after obsolescence is the same as for the separated large firm, i.e., $(1+\delta)\,\Lambda\,X$ for $t \geq \mathcal{T}_Y$. We denote the integrated firm's equity and firm values when the growth option has been preempted by E_I° and V_I°, with $\underline{X}_I^\circ$ again being the corresponding default threshold selected by shareholders. Observe next that for a given coupon payment there is no difference between Integration and Non-Integration after the option has become obsolete due to preemption. Therefore, the analytic expressions from Lemma 3 directly apply to this organizational design, which yields the next result.

Lemma 5 *Given the integrated firm's initial coupon choice C_I and the current value of cash flow X, total firm and equity values are for all $t \geq \mathcal{T}_Y$ and $X \geq \underline{X}_L^\circ$ given by (20) and (21), and the equity value-maximizing default threshold is given by (23) where subscripts L are replaced by subscripts I.*

Working backwards, the value of the integrated firm prior to exercise or obsolescence equals the expected present value of the levered firm values in three regions: (i) before investment and preemption, (ii) after investment, and (iii) after preemption. We denote the integrated firm's equity and firm values at time zero by E_I and V_I. Moreover, let $\underline{X}_I$ denote the default threshold in region (i) and the first time for X to touch this threshold from below by $\mathcal{T}_D$, while $\overline{X}_I$ is the investment threshold for moving from region (i) to region (ii) at time $\mathcal{T}_G$.

For brevity, we defer the derivations of firm value and equity value to the Appendix. For example, the derivation of firm value involves the same steps as outlined by (24) and (25). The contingent claim values and value-maximizing decisions under Integration are collected in the next proposition.

Proposition 3 *Given the current value of cash flow X, the integrated firm's total value equals for all $X \in \left(\underline{X}_I, \overline{X}_I\right)$ and $t < \mathcal{T}_Y$:*

$$V_I(X) \;=\; AIP_I(X) \,+\, G_I(X) \,+\, NTB_I(X)\;, \tag{36}$$

where the value of assets in place, AIP_I, is given by:

$$AIP_I(X) \;=\; \Lambda\, X \,+\, \frac{\rho}{r+\rho-\mu}\,\delta\,\Lambda\left[X - \widehat{\Sigma}(X)\,\overline{X}_I\right] \,+\, \gamma\,\Lambda\,\overline{X}_I\widehat{\Sigma}(X), \tag{37}$$

the value of the growth option for the large firm is given by:

$$G_I(X) \;=\; \left[\pi\,\Lambda\,\overline{X}_I \,-\, \kappa\right]\widehat{\Sigma}(X), \tag{38}$$

and the value of net tax benefits, NTB_I, is given by:

$$\begin{aligned} NTB_I(X) \;=\; & \frac{\tau\, C_I}{r}\left(1 \,-\, \widehat{\Delta}(X) \,-\, \widehat{\Sigma}(X)\left(\frac{\overline{X}_I}{\underline{X}_I^+}\right)^{\vartheta'} \,-\, \frac{\rho}{r-\mu'}\,\widehat{\Psi}(X)\right) \\ & -\alpha\left(\widehat{\Delta}(X)\,\Lambda\,\underline{X}_I \,+\, \widehat{\Sigma}(X)\,(1+\pi)\,\Lambda\,\underline{X}_I^+\left(\frac{\overline{X}_I}{\underline{X}_I^+}\right)^{\vartheta'} \,+\, \frac{\rho}{r-\mu'}\,\Lambda\,\underline{X}_I^\circ\,\widehat{\Psi}(X)\right)\;. \end{aligned} \tag{39}$$

The value of the integrated firm's equity equals for all $X \in \left(\underline{X}_L, \overline{X}_I\right)$ and $t < \mathcal{T}_Y$:

$$\begin{aligned} E_I(X) \;=\; & \left(\Lambda\, X \,-\, \frac{(1-\tau)\,C_I}{r}\right) \,+\, \frac{\rho}{r+\rho-\mu}\,\delta\,\Lambda\left[X - \widehat{\Delta}(X)\,\underline{X}_I - \widehat{\Sigma}(X)\,\overline{X}_I\right] \\ & + \left[(\pi+\gamma)\,\Lambda\,\overline{X}_I - \kappa\right]\widehat{\Sigma}(X) \,-\, \frac{\rho}{r-\mu'}\left((1+\delta)\,\Lambda\,\underline{X}_I^\circ \,-\, \frac{(1-\tau)\,C_I}{r}\right)\widehat{\Psi}(X) \\ & -\widehat{\Delta}(X)\left(\Lambda\,\underline{X}_I \,-\, \frac{(1-\tau)\,C_I}{r}\right) \,-\, \widehat{\Sigma}(X)\left((1+\pi+\gamma)\,\Lambda\, X \,-\, \frac{(1-\tau)\,C_I}{r}\right)\left(\frac{\overline{X}_I}{\underline{X}_I^+}\right)^{\vartheta'}, \end{aligned} \tag{40}$$

where the stochastic discount factors for default and investment by the large firm are given by:

$$\widehat{\Delta}(X) \;=\; \frac{X^\xi\overline{X}_I^\vartheta - X^\vartheta\overline{X}_I^\xi}{\underline{X}_I^\xi\overline{X}_I^\vartheta - \underline{X}_I^\vartheta\overline{X}_I^\xi}\;, \qquad \textit{and} \qquad \widehat{\Sigma}(X) \;=\; \frac{\underline{X}_I^\xi X^\vartheta - \underline{X}_I^\vartheta X^\xi}{\underline{X}_I^\xi\overline{X}_I^\vartheta - \underline{X}_I^\vartheta\overline{X}_I^\xi}\;, \tag{41}$$

the adjusted growth rate is $\mu' = \vartheta' \mu + \frac{1}{2} \vartheta' (\vartheta' - 1) \sigma^2$, *and the terms related to preemption risk are*

$$\widehat{\Psi}(X) \;=\; \left(\frac{X}{\underline{X}_I^{\circ}}\right)^{\vartheta'} - \widehat{\Delta}(X)\left(\frac{\underline{X}_I}{\underline{X}_I^{\circ}}\right)^{\vartheta'} - \widehat{\Sigma}(X)\left(\frac{\overline{X}_I}{\underline{X}_I^{\circ}}\right)^{\vartheta'} \quad , \tag{42}$$

and where ϑ', ξ, *and* ϑ *are given in (4), (15), and (32), respectively. Finally, the optimal (firm value-maximizing) coupon choice solves* $\max_{C_I} V_I(X)$, *the optimal (equity value-maximizing) investment threshold solves* $\partial E_I(X)/\partial X|_{X=\underline{X}_I} = \partial E_I^+(X)/\partial X\big|_{X=\underline{X}_I}$, *and the optimal (equity value-maximizing) default threshold solves* $\partial E_I(X)/\partial X|_{X=\underline{X}_I} = 0$.

Proposition 3 presents closed-form solutions when cash flows from assets in place are affected by both diffusion risk (i.e., cash flow uncertainty) and jump risk (i.e., obsolescence/preemption uncertainty). In the Integrated case, the value of the firm can be decomposed into three main parts, the value of assets in place, $AIP_I(X)$, the value of the growth option, $G_I(X)$, and the value of net tax benefits, $NTB_I(X)$, which explains (36). Clearly, a major difference from Proposition 2 is that in the Integrated case the firm's value includes also the value of the growth option, $G_I(X)$, so that capital structure decisions affect the value of the growth option in the Integrated case.

Understanding the expressions for each value component is best accomplished by comparison to Propositions 1 and 2. First, the expression for assets in place value, $AIP_I(X)$, is analogous to $AIP_L(X)$ from Proposition 2, with the only difference being that the state prices for the cannibalization terms must now take into account the probability that the integrated firm will default and the option will be destroyed prior to exercise. This is reflected in the replacement of $(X/\overline{X}_S)^{\xi}$ in (27) with $\widehat{\Sigma}(X)$ in (37). The probability of default by the large firm was not relevant for the value of assets in place in the Non-Integrated case since default had no effect on the growth option, and therefore the implications of cannibalization from its exercise or preemption were the same for the new owners of the assets in place after default as for the original owners prior to default. Second, $NTB_I(X)$ is analogous to $NTB_L(X)$ from Proposition 2, with the state prices simply adjusted for the different exercise and default decisions taken by the integrated firm.

Finally, the expression for $G_I(X)$ is analogous to $G_S(X)$ in Proposition 1, with the only difference being a two-sided state price $\widehat{\Sigma}(X)$ to account for the possibility that the integrated firm will default prior to exercise and the growth option will be lost. As a result, the integrated firm's option in (38) will always be worth less than it is for the separated, small firm in (12), so long as $C_I > 0$. To understand this, first consider a comparison of the two while holding the exercise threshold constant. Note first that the two-sided investment claim converges to the one-sided investment claim as $\underline{X}_L$ goes to zero; that is,

$$\lim_{\underline{X}_L \downarrow 0} \frac{\underline{X}_I^{\xi} X^{\vartheta} - \underline{X}_I^{\vartheta} X^{\xi}}{\underline{X}_I^{\xi} \overline{X}_I^{\vartheta} - \underline{X}_I^{\vartheta} \overline{X}_I^{\xi}} \;=\; \left(\frac{X}{\overline{X}_I}\right)^{\xi} \quad , \tag{43}$$

where the term on the right-hand side corresponds to the state price in (12) if we replace $\overline{X}_I$ by $\overline{X}_S$. Second, the first derivative of the two-sided investment claim w.r.t. $\underline{X}_I$ is given by:

$$\frac{\partial}{\partial \underline{X}_L}\left(\frac{\underline{X}_I^{\xi} X^{\vartheta} - \underline{X}_I^{\vartheta} X^{\xi}}{\underline{X}_I^{\xi}\overline{X}_I^{\vartheta} - \underline{X}_I^{\vartheta}\overline{X}_I^{\xi}}\right) = (\xi - \vartheta)\,\frac{\left(X^{\xi}\overline{X}_I^{\vartheta} - X^{\vartheta}\overline{X}_I^{\xi}\right)}{\left(\underline{X}_I^{\xi}\overline{X}_I^{\vartheta} - \underline{X}_I^{\vartheta}\overline{X}_I^{\xi}\right)^2}, \tag{44}$$

which is negative as the numerator on the right-hand side is negative if $X^{\xi-\vartheta} < \overline{X}_I^{\xi-\vartheta}$, which is clearly the case since $X < \overline{X}_I$ at time zero. Optimization, of course, implies that exercise will take place at different threshold levels under the two different organizational forms (i.e., $\overline{X}_I \neq \overline{X}_S$), which, as we will see, increases the wedge in option values between the two organizational forms.

The differences in equilibrium option values between the two organizational forms will clearly be driven by interactions among the firms' strategic decisions, i.e., their debt coupon choices and investment and default thresholds. Coupon choices will be different across the two cases since the integrated firm will issue debt at time zero which reflects both the debt capacity of assets in place and the *expected* debt capacity of the assets created upon exercise of the growth option, while the large firm in the Non-Integrated case will consider only the debt capacity of assets in place. In both cases, however, the firms consider possible future reductions in the debt capacity of the assets in place due to cannibalization. In addition, the firms' capital structure choices affect option value in two ways: by affecting the optimal exercise trigger (with a delay due to the debt overhang effect in the Integrated case, but not in the Non-Integrated case), and by differences in how default by the owner of the assets in place affects the value of the option (default by the existing firm destroys the option in the Integrated case, but has no effect in the Non-Integrated case). Finally, the strategic investment decisions affect growth option value directly, but also affect assets in place value through cannibalization, which is taken into account by the integrated firm in choosing its exercise threshold, but not by the small firm in the Non-Integrated case.

4 Results and Implications

To illustrate some of the model's results in more detail, we now provide a number of numerical simulations in which we determine which organizational form maximizes the joint time zero value of the two corporate activities. Our primary focus in this section is on the implications of cannibalization costs, risk types, and capital structure determinants.

4.1 Cannibalization Implications

We start by considering the effect of changes in the main cannibalization parameter, γ, in a baseline environment in which cash flows start at $X = \$20$, the risk-free interest rate is $r = 7\%$, the growth rate of cash flows is $\mu = 1\%$, the volatility of cash flows is $\sigma = 30\%$, the risk of obsolescence is $\rho = 10\%$, the corporate tax rate is $\tau = 15\%$, the proportional cost of bankruptcy is $\alpha = 30\%$, the investment factor is $\pi = 100\%$, the investment cost is $\kappa = \$225$, and the cannibalization effect from obsolescence is $\delta = 0\%$.[6]

In each equilibrium configuration, the total joint value of the projects can be broken down into three main categories: the cash flow value of assets in place, the cash flow value of the growth option, and the value of net tax benefits (i.e., tax shields less expected costs of financial distress). The organizational form that best balances these three sources of value will be optimal.

First consider a case when the cost of cannibalization from option exercise, γ, is small in absolute value, i.e., close to zero. In this case the exercise of the growth option has little effect on the value of assets in place. Since the existing firm has significant cash flows from assets in place, it will be optimal to carry a significant amount of debt. However, due to the well known debt overhang effect, the existence of this debt in the Integrated case will significantly alter the firm's chosen exercise policy. In particular, equityholders will tend to exercise "too late" since some of the value of option exercise will confer to debtholders. In turn, anticipating this effect the firm will issue less debt, reducing the value of net tax benefits. On the other hand, in the Non-Integrated form the option exercise time is chosen in an environment that is independent from the assets in place and resulting agency conflict with debtholders. It is therefore likely that the Non-Integrated organizational form maximizes the value of the growth option. In addition, the large firm does not have to reduce its debt level to avoid the overhang effect, which increases the value of net tax benefits related to assets in place. Further enhancing this effect is the fact that the small firm in the Non-Integrated case will be able to choose an optimal debt level for the new assets at the time of exercise. Thus, Non-Integration is likely to best balance the three sources of value, namely the values of assets in place, growth option, and net tax benefits.

Notably, as γ decreases the time of exercise starts having a significant effect on the value of assets in place. Thus, an element of joint profit maximization becomes important in balancing the value of the growth option against the future value of assets in place. Since the small firm in the Non-Integrated case completely ignores this effect given that there is no joint profit maximization by design, the small firm's exercise policy imposes increasingly larger costs on the large firm's assets in place value as γ falls. Thus, there will likely always exist a cutoff level of gamma, say γ^*, such that Integration will be the optimal organizational form for $\gamma < \gamma^*$, while Non-Integration will be optimal for $\gamma > \gamma^*$. (This conjecture is borne out in every numerical simulation we have attempted.)

[6] While the base case parameter choices could be motivated in more detail, we omit this for the sake of brevity and note that the model's results and implications only vary quantitatively but not qualitatively with parameters.

To see this effect, consider Figure 1 below. Panel (a) of the figure graphs the optimal exercise time for each organizational form (the solid line in all figures corresponds to the Integrated form, while the dashed line corresponds to the Non-Integrated form) as a function of γ given the base parameters provided above. Note that the optimal exercise time in the Non-Integrated case is invariant to γ—the small firm ignores the effect of its exercise on the the large firm's assets in place. Also, as expected, the Integrated firm responds aggressively to changes in γ, exercising much later when the cannibalization effect is large.

[Insert Figure 1 here]

Panel (b) of the figure graphs the time zero value of assets in place as a function of γ. Consistent with the results in Panel (a), the value of assets in place is much more sensitive to γ in the Non-Integrated case since the small firm's exercise policy does not react, and the assets in place are subjected directly to changes in cannibalization. In the Integrated case the firm's optimal tradeoff between the value of the option and the effect on assets in place dampens the relationship. Overall, the gap in assets in place value between the two forms grows quickly as γ falls.

Panel (c) of the figure graphs the time zero value of the growth option as a function of γ. Consistent with Panel (a), in the Non-Integrated case the option value is insensitive to γ (the time of exercise is the main variable that affects option value). However, in the Integrated case option value is highly sensitive to γ as the exercise time is adjusted to account for cannibalization. In particular, option value in the Integrated case rises quickly as γ becomes smaller in absolute value since the integrated firm no longer has to worry so much about cannibalization and its exercise policy gets closer to that of the small firm in the Non-Integrated case.

Panel (d) of the figure graphs the time zero value of total net tax benefits. Total tax benefits are always higher in the Non-Integrated case since capital structure for the new assets is set at the time of option exercise, which best maximizes the associated net tax benefits. The difference between the curves is not particularly sensitive to changes in γ (relative to the sensitivity of assets in place value and growth option value), and therefore does not contribute much to the comparative static.

Finally, Panel (e) compares total time zero value of the two projects across the different organizational forms (i.e., it is the sum of assets in place value, option value, and value of net tax benefits from the three prior graphs). As discussed above, the two curves cross at the critical cannibalization value $\gamma^* \approx -17.5\%$ with Integration being optimal for all lower γ and Non-Integration being optimal for all higher γ. In comparing the three prior graphs, it is clear that this is being driven mainly by relative changes in assets in place and option values since the two organizational forms place relatively more/less weight on jointly or separately optimizing assets in place and option values. For low γ Integration best protects assets in place value, while for high γ this effect is less important, and the greater option value and net tax benefits of the Non-Integrated form dominate.

4.2 Risk Implications

To investigate the effects of other parameters on the organizational design choice, we use the clear-cut γ^* result as a baseline characterization of the solution, and study the comparative statics of γ^* with respect to the remaining parameters. First consider ρ, which measures the risk of obsolescence, and σ, which measures the underlying uncertainty of the cash flows. Figure 2 below provides two equilibrium "maps" which plot the optimal organizational form as a function of γ and obsolescence risk ρ (panel (a)), or γ and cash flow risk σ (panel (b)), holding all other parameters constant at their base levels. In this and all proceeding figures, the shaded area of each map represents the part of the parameter space for which Non-Integration is the optimal organizational form, and the white part of the map represents that part where Integration is optimal.

[Insert Figure 2 here]

First consider panel (a). The existence of the cutoff γ^* is clearly verified for all ρ considered in the map, from zero to 50%. There is also a clear effect that γ^* is monotonically increasing in ρ. In other words, Integration is more likely to be optimal at high ρ than at low ρ. To understand this, first consider the optimal exercise policy as ρ increases. As the probability of preemption becomes higher, the firm holding the growth option must speed up exercise significantly to maintain the value of the option. The small firm in the Non-Integrated case always exercises sooner than the Integrated firm, which waits in order to avoid excessive cannibalization and because of debt overhang. See Panel (a) of Figure 3 below for an illustration of this effect when $\gamma = -0.175$. Note that the exercise times in the two cases decrease in ρ similarly, but the small firm in the Non-Integrated case always exercises significantly earlier than the Integrated firm.

[Insert Figure 3 here]

Now consider Panel (b) of Figure 3, which plots assets in place value as a function of ρ. Recall that assets in place value will be higher the longer the owner of the option waits to exercise because of a lower cannibalization effect, and thus the value of assets in place will always be higher in the Integrated case (the solid line is always higher than the dashed line). More importantly, though, note that the gap in the value of assets in place grows significantly as ρ rises despite the fact that the difference in exercise times does not grow very quickly. This is because the value of assets in place is increasing and *concave* in the exercise time—i.e., the delay caused by moving from the

Non-integrated to the Integrated form has a much stronger impact on assets in place value when the exercise time is sooner (closer to the initial value of X). As a result, the Integrated form is more likely to dominate at high ρ values as its ability to better preserve assets in place value becomes more important. At lower ρ the cost to assets in place from the small firm's earlier exercise policy is not so high, so its higher option value and net tax benefits tend to dominate.

The result that Non-Integration is more likely when preemption risk is lower may seem somewhat counter-intuitive, as many argue that small firms are better able to respond in highly dynamic markets. This may be true, but our results indicate that when the source of high uncertainty is the risk of preemption by third parties, a small innovator's behavior may impose excessive costs on incumbent firms, so that it could be optimal for them to be absorbed by existing players in the market despite the negative impact on their own value. The resulting empirical implication is then that major innovative advances are more likely to arise within specialized, small firms when the ideas are so novel that preemption is unlikely, but ideas that are more aggressively contested by competing innovators might be more often incubated within existing firms. This is not because the existing firm is more able to invest aggressively to ward off competition (which may also be true – see, e.g., Mathews and Robinson (2008)), but because the incumbent firm's own assets are better protected.

Next consider panel (b) of Figure 2. Again, the existence of the cutoff γ^* is verified for all values of σ, the volatility of cash flows. Also, γ^* again varies monotonically, but this time is clearly decreasing in σ. To understand this, consider the effect of increasing σ on the growth option. Option theory tells us that as σ rises the option's value increases, and exercise should occur later (see panel (c) of Figure 3 for an illustration). The latter implication means that higher σ states will be those where the small firm's earlier exercise choice (because of the lack of debt overhang and lack of concern for cannibalization) has less of a negative impact on the value of assets in place (since, as noted above, assets in place value is concave in exercise time). Thus, assets in place are worth more in the Non-Integrated case in *relative* terms at higher σ, implying that Non-Integration is more likely to be optimal in environments with greater underlying cash flow risk (see panel (d) of Figure 3 for an illustration). This is consistent with the conventional wisdom that small firms are more innovative in uncertain environments.

The above results imply an interesting dichotomy wherein the effect of risk on organizational design depends strongly on the type of risk being considered. Whereas greater cash flow risk, such as uncertainty about market size, tends to favor the operation of innovative projects in independent firms, greater preemption risk instead tends to favor the operation of such projects within existing larger firms. This dichotomy should prove useful for empirical investigations of why innovation tends to occur in different organizational structures across different markets and/or time periods.

4.3 Capital Structure Implications

Next consider the two parameters that most directly measure the importance of capital structure effects, namely τ, the corporate tax rate, and α, the magnitude of bankruptcy costs. Figure 4 provides equilibrium maps for these parameters. First consider panel (a). Here there is a very clear pattern in that γ^* decreases quickly in τ. An increase in τ clearly has multiple effects – it directly reduces after-tax cash flows, while it at the same time makes capital structure decisions and their associated value implications more important. The fact that the Non-Integrated form becomes more dominant as τ rises comes mostly from the latter effect. Specifically, the net tax benefits of debt rise faster in τ for the Non-Integrated form than for the Integrated form, and specifically because the Non-Integrated form is better able to utilize the debt capacity of the growth option. When tax rates rise option exercise is delayed in both forms, but the timing of pre-tax cash flows differs across forms because in case of Integration debt overhang creates additional delay, which grows with τ.

[Insert Figure 4 here]

Next consider panel (b) of Figure 4, which shows the effect of the bankruptcy cost parameter, α. Here, γ^* is increasing in α. The effect of α is more straightforward than the effect of τ since there is no confounding effect on overall profitability – i.e., α just directly impacts the net tax benefits the firm can enjoy. The direction of the effect has essentially the same intuition as the effect of τ, in that greater bankruptcy costs decreases the importance of net tax benefits as a source of value, and since protecting that value was one reason for choosing Non-Integration, that choice is less likely to be optimal when the available value shrinks.

Unlike the tax rate, bankruptcy costs do not directly affect cash flows and hence variations in α provide a better gauge for how debt overhang influences organizational design. All else equal, lower bankruptcy costs imply higher optimal coupon payments and, even if incorporated into the trade-off problem, produce on the margin more overhang costs that are a disadvantage of integration. Consistent with this intuition, the critical cutoff γ^* decreases at an increasing rate when α declines (i.e., Non-Integration is also more likely for lower bankruptcy costs because they are associated with higher debt overhang costs under Integration).

4.4 Additional Implications

In the base specification above, for simplicity we assume that when the growth option becomes obsolete, there is no cannibalization effect on the assets in place (i.e., $\delta = 0$). Panel (a) of Figure 5 below provides an equilibrium map showing the effect of including a cannibalization effect due to the operation of a competing asset by a third party firm or firms.

[Insert Figure 5 here]

The map clearly shows that the cutoff for the main cannibalization parameter, γ^* grows as the obsolescence cannibalization parameter becomes significant (δ falls). In other words, Non-Integration is more likely to be optimal when obsolescence imposes a large cost on the assets in place. To understand this, first consider starting from the base case of $\delta = 0$. In this case, the Integrated firm will clearly exercise later than the Non-Integrated firm because of debt overhang and the desire to avoid the main cannibalization effect (γ). Thus, in relative terms the Non-Integrated form imposes a significant cost on the value of assets in place, while the Integrated form imposes a significant cost on option value. However, the integrated firm starts exercising the option earlier as δ becomes larger in absolute value in order to avoid preemption and thus preserve option value (it is better to get the benefits of investment despite the cannibalization cost γ than to suffer preemptive cannibalization δ with no offsetting payoff). This brings the exercise times closer and shrinks the gap in assets in place and option values, with the former having a larger effect. Since the gap in assets in place values shrinks faster, Non-Integration is more likely to be optimal (it still offers greater option value and net tax benefits, without imposing too great a cost on assets in place).

The size of the growth option's potential payoff relative to the size of assets in place, π, is also an important determinant of the optimal organizational form. Panel (b) of Figure 5 provides the relevant equilibrium map. As expected, the larger is π the more likely it is that Non-Integration is optimal, as this form best protects the value of the option and its associated net tax benefits, which become more important as π grows.

The remaining parameters have relatively little effect on the organizational design decision. Figure 6 below provides equilibrium maps for the investment cost, κ, and the growth rate, μ. While the effects are not large, Non-Integration is more likely to be optimal the higher are both κ and μ. A higher κ value makes the owner of the option wait longer to exercise in either organizational form, and because of the concavity of the value of assets in place with respect to the exercise date, the value of those assets rises faster with κ in the Non-Integrated case. An increase in μ not only enhances the option payoff, which makes maximizing pure option value more important, but it also induces the owner of the option to exercise it sooner because the opportunity cost of waiting increases with the larger rate of forgone cash flows, which leads to higher costs of cannibalization from option exercise. In addition, however, a higher μ raises the value of assets in place and hence induces the integrated form to optimally issue more debt at time zero, which increases the cost of debt overhang. Taken together, these effects produce a γ^* profile that declines with μ.

[Insert Figure 6 here]

5 Extensions

In this section, we study two extensions to the main model. First, we consider a hybrid organizational form that is relatively common in practice, namely a financial alliance, and examine how it alters the predictions from the main model in Section 4. Second, we examine an extension that enhances the financial flexibility of the large, integrated firm and revisit the equilibrium maps for identifying the regions in which Non-Integration and Integration are optimal within the extended model.

5.1 Financial Alliances

Thus far, our analysis has assumed two possible organizational arrangements, complete integration or complete non-integration. In reality, there are a multitude of possible organizational design choices with these two arrangements at either end of a continuum, and hybrid forms such as joint ventures and alliances in between. It is therefore natural to ask whether such a hybrid form could dominate the two extreme forms considered above. In this section we consider one particular such hybrid form, defined by a contractual arrangement between two separate organizations, which we refer to as a financial alliance to distinguish it from other possible types of alliances.[7]

A defining characteristic of many joint ventures and alliances is a contract that specifies the rights of each involved party with respect to exploiting any innovations arising from the relationship. For example, contracts may specify rights to market new products in specific geographical regions, or in particular forms. These agreements often come in the form of licensing arrangements. In the context of our model, such arrangements are particularly interesting because they will likely affect the parties' incentives with respect to option exercise timing and capital structure. We thus investigate whether a licensing-type contract between two separate organizational forms can help in providing a superior tradeoff between the three sources of value in our setting: assets in place value, pure growth option value, and net tax benefits.

To model the alliance, it is easiest to start from our model's Non-Integrated case. In this context, a financial alliance involves a licensing contract that stipulates a proportion, ℓ, of the future cash flows of the growth option that are pledged to the large firm. We assume that the small firm retains full decision rights over option exercise timing and its own capital structure, as

[7] For example, a strategic product market alliance could be an arrangement that has a direct controlling effect on the extent of cannibalization, γ, which it could be natural to assume might vary across different organizational forms.

well as full responsibility for funding the exercise cost. Intuitively, siphoning off more of the benefit from exercising the option to the large firm will cause the small firm to exercise later, which helps protect the value of assets in place. Furthermore, since the small firm retains the right to choose the exercise time, this arrangement avoids imposing the cost of debt overhang that would arise with a switch to a fully Integrated form. On the other hand, the delay in exercise timing will decrease the pure value of the growth option. The licensing contract also affects net tax benefits since the small firm will optimally take on less debt at exercise, while the large firm will take on more debt at time zero in anticipation of receiving the extra cash flows in the future. This will tend to result in a lower overall value of net tax benefits since the contract moves the cash flow allocation more toward the Integrated form, which inherently has less capital structure flexibility. Thus, the optimality of such a licensing contract depends on whether the two benefits (protecting assets in place value and avoiding overhang costs) can outweigh the two costs (reduced growth option value and net tax benefits).[8]

Re-solving the model with the addition of the parameter ℓ is relatively straightforward. In the valuation equations for the small firm in Section 3.1, every instance where the scaling parameter π appears would be changed to $(1-\ell)\,\pi$. For example, the small firm's post exercise value, previously (2) in the Non-Integrated case, becomes

$$V_S^+(X;\ell) \;=\; (1-\ell)\,\pi\,\Lambda\,X \;+\; \frac{\tau\,C_S^+}{r}\left(1-\left(\frac{X}{\underline{X}_S^+}\right)^{\vartheta'}\right) \;-\; \alpha\,(1-\ell)\,\pi\,\Lambda\,\underline{X}_S^+\left(\frac{X}{\underline{X}_S^+}\right)^{\vartheta'} \;, \tag{45}$$

while its objective function remains the same since it funds the entire exercise cost, κ:

$$V_S(X;\ell) \;=\; \sup_{\mathcal{T}_G} \mathrm{E}\left[\,1_{\mathcal{T}_G<\mathcal{T}_Y}\, e^{-r\mathcal{T}_G}\left(V_S^+(X_{\mathcal{T}_G};\ell) \;-\; \kappa\right)\right] \;. \tag{46}$$

Similarly, for the large firm's valuation equations in Section 3.2, everywhere that the cannibalization term γ appears would be changed to $(\gamma \,+\, \ell\,\pi)$ since the extra cash flows to the large firm following option exercise occur in exactly the same states as the reduction in cash flows due to cannibalization (and thus can equivalently be seen as an adjustment to the cannibalization parameter from the large firm's perspective).

To illustrate the impact of the licensing parameter ℓ, we first simulate the model at the same base parameters used previously, and investigate the impact on decisions and value as ℓ is adjusted (thinking of ℓ as an exogenous parameter for now). Panel (a) of Figure 7 plots the small firm's optimal exercise timing as a function of ℓ. Note that this and all remaining panels of the figure

[8]Note that an alternative alliance contract could specify a fee paid to the large firm by the small firm at the time of exercise. Carefully choosing the level of the fee would also have the effect of calibrating the small firm's exercise decision to a joint optimum to protect assets in place and avoid overhang costs. It may also distort capital structure decisions less than the licensing alliance considered here. However, we have chosen not to focus on that alternative since it may be more difficult to write or enforce, and is not often observed in reality.

also show the equivalent value for the Integrated firm (represented by the blue line), whose choices and values are not affected by ℓ. As expected (and as is easy to see analytically using equation (14)), increasing the proportion of cash flows given to the large firm delays the small firm's exercise in equilibrium. This will clearly increase the value of assets in place (see panel (b) of the figure) which, as noted previously, is increasing and concave in exercise time. At the same time (panels (c) and (d)), the value of the growth option and of total net tax benefits fall. Panel (e) puts all of these effects together, and shows that overall joint firm value is initially increasing in ℓ as the effect on assets in place dominates, but at some point the erosion of option value and net tax benefits becomes dominant. As a result, joint value is a concave function of ℓ (a result that is robust to every parameterization we have used). This implies that there will generally exist an optimal licensing proportion, ℓ^*, that best balances the benefits and costs of the alliance. Since a licensing proportion of $\ell = 0$ is the same as our Non-Integrated case, this means that an optimally chosen financial alliance will generically be preferred to straight Non-Integration. However, it might or might not induce better joint profit maximization than the Integrated form. In particular, an optimally structured financial alliance will clearly dominate the Integrated form in cases where Non-Integration was already optimal (i.e., the shaded regions of the equilibrium maps above). In addition, our simulations show that a financial alliance with the optimal licensing proportion can often dominate Integration even in the non-shaded portions of the equilibrium maps in the previous section.

[Insert Figure 7 here]

We next investigate how the optimal licensing contract, ℓ^*, varies with the model's other parameters. These comparative statics should provide direct empirical implications for studies of alliance structuring. Figure 8 below plots the optimal licensing fraction against the four parameters that have been found to have the greatest influence. To understand these comparative statics, first recall that a financial alliance as modeled here is essentially an intermediate organizational form between the extremes of Non-Integration and Integration. One of the key differences between these extremes arises from the differing allocation of control over the exercise timing of the option. When the small firm has control it tends to exercise sooner to maximize option value, while when the large firm has control it tends to exercise later due to debt overhang and a desire to protect the value of assets in place. The licensing contract modeled here serves to effectively "bridge the gap" between these two extremes by coordinating on an intermediate exercise time that better maximizes joint profits (while avoiding the costs of debt overhang).

Now consider Panel (a) of the figure, which shows the effect of γ, the cannibalization cost due to option exercise, on ℓ^*. Unsurprisingly, the larger is γ in absolute value, the larger is the optimal licensing proportion. As the cannibalization effect of option exercise grows, it becomes more important to protect the value of assets in place, which is accomplished by pushing the exercise time more toward that resulting from joint profit maximization of the large firm in the Integrated case. A larger licensing proportion accomplishes exactly that, i.e., it moves the hybrid organizational form closer to Integration, without incurring costs of debt overhang to delay exercise, and therefore better protects the value of assets in place.

Panel (b) of the figure shows the effect of τ, the corporate tax rate. The larger is τ, the smaller is the optimal licensing proportion. As τ grows, capital structure effects become more important, which, when considering just the extremes, tends to favor Non-Integration over Integration due to its greater capital structure flexibility. In addition, the exercise threshold increases with τ and hence protecting the large firm's assets in place is less important. Taken together, an increase in τ makes the negative effect of licensing on net tax shield value more important, and thus the alliance is optimally pushed more toward the Non-Integrated form by choosing a lower licensing proportion.

[Insert Figure 8 here]

Panel (c) of the figure plots the effect of changes in π, the size of the growth option. Clearly, an increase in π tends to decrease the optimal licensing proportion. The logic here is based again on the relative importance of protecting assets in place versus preserving option value: as π increases it becomes relatively more important to preserve option value because the firm's assets in place are normalized to one, and since a move toward the Integrated form erodes option value by pushing the exercise time later, a lower proportion is chosen despite the cost to assets in place value.

Finally, panel (d) shows the effect of the obsolescence cannibalization parameter, δ. In this case, the optimal licensing proportion falls as δ rises in absolute value. As δ becomes large, the large firm prefers earlier exercise times to avoid experiencing cannibalization with no offsetting cash flows, so the small firm's bias toward an early exercise time becomes more in line with joint profit optimization. In other words, the change in δ does not affect the optimal exercise time in the Non-Integrated case, but it makes it significantly earlier in the Integrated case, so a smaller ℓ is sufficient to optimally bridge the gap between these preferred times.

The remaining parameters have less of an influence on the optimal licensing proportion. Figure 9 below illustrates this phenomenon for cash flow uncertainty, σ, and obsolescence risk, ρ. In contrast to the results above where these parameters had strong and opposite effects on the cutoff cannibalization level γ^*, in the case they have similar, and very minor, effects on the optimal alliance contract ℓ^*. To understand this, note that changes in both σ and ρ will have similar effects on growth option value and exercise policy no matter who controls the option. In particular, an increase in σ

increases the value of the option and induces later exercise times whether the option is controlled by the small firm (as in the Non-Integrated case), or by the large firm (as in the Integrated case). Since these exercise times move together with changes in σ, and the gap between the optimal exercise times from the two firms' perspectives does not change much, the optimal licensing proportion (which, as noted above, is essentially set to ameliorate this gap) is not significantly changed. Similarly, an increase in ρ tends to reduce option value and induce earlier exercise times no matter who controls the option, without significantly changing the size of the gap in exercise times.

[Insert Figure 9 here]

5.2 Alternative Financing Arrangements

As has been noted, one might be tempted to conclude that the Non-Integrated form often dominates the Integrated form largely because it has inherently more capital structure flexibility in our base case analysis. It is therefore natural to examine the extent to which the optimal choice of organizational form and the resulting comparative statics results might depend upon the assumption that the Integrated form cannot issue debt at the exercise time of the innovation option. In this section we analyze the effect of an alternative financing arrangement in the Integrated case, in which the integrated, large firm is able to follow a similar capital structure policy as the separated, small firm in the Non-Integrated case at the time of option exercise.

Specifically, at the time of option exercise in the Integrated case we grant the integrated firm the option to recapitalize with respect to the new collateral pool obtained by exercising the option (i.e., π), but not with respect to the existing collateral pool that we normalized to one (i.e., assets in place). In addition to the time zero debt with coupon C_I, we assume the integrated firm issues a second, time $\mathcal{T}_G$ debt tranche with coupon payments C_I^+ specified as in (6). Specifying this amount of debt (i.e., the amount that would be chosen by an all-equity stand-alone firm for this set of assets at the time of exercise) removes conflicts of interest across debt classes which are not present in the Non-Integrated case, but would otherwise exist in the Integrated case and be a disadvantage of integration. Moreover, we need to specify debt priority in bankruptcy, which primarily affects our result through the floatation value of the new debt. We assume that the two classes of debt receive equal treatment in bankruptcy.[9] We continue to assume that the exercise time is chosen by the firm's

[9]That is, the relative weights implied by the two debt coupons apportion the firm's recovery value according to an equal priority (*pari passu*) rule. If the new class if debt is junior to the time zero debt, then its floatation value is lower and so are the benefits from the additional financial flexibility for the integrated firm. We use equal priority for this model extension because it presents a more significant deviation from the base case model and because later debt issues are rarely senior to earlier debt issues unless the firm undergoes, e.g., a formal restructuring with debtor-in-possession financing. For an analysis of optimal priority structure, see, e.g., Hackbarth and Mauer (2010).

equityholders to maximize their own value plus the value of the new debt. These assumptions can be modified or relaxed, but they are made to be as consistent as possible with the treatment of the separated, small firm in Lemma 1 and Proposition 1. Note that under this alternative specification, the large, separated firm does not have an option to recapitalize after its time zero debt choice.[10]

Intuitively, this model extension attenuates debt overhang concerns because the integrated firm will choose less debt at time zero and can use the proceeds from the later debt issue to fund a fraction of the exercise cost. Hence the integrated firm maximizes more the pure growth option value in its initial financing choice. However, this leads endogenously to a larger cannibalization effect on its asset in place value. Furthermore, the wedge in net tax benefits across forms seen in Panel (d) of Figure 1 essentially disappears, meaning net tax benefits are no longer an overall driver of organizational design. It is therefore important to re-examine some of the key equilibrium maps from Section 4 to see whether the behavior of the critical cutoff value for γ^* would be materially altered under this setup.

[Insert Figure 10 here]

Figure 10 provides four equilibrium maps, which trace again the critical value γ^* as a function of various model parameters. In particular, we re-examine for this model extension the optimal organizational form as a function of γ and ρ (panel (a)), γ and σ (panel (b)), γ and α (panel (c)), and γ and τ (panel (d)), holding all other parameters constant at their base levels. While the cutoff for the cannibalization parameter to provoke integration is lower in absolute value, the comparative statics of γ^* are qualitatively unchanged relative to the base case model. In other words, the predictions of γ^*'s directional behavior from the main model are unaffected by giving the Integrated form more financial flexibility. The fact that the level of γ^* at the base parameter values is now lower in absolute value makes intuitive sense because granting the Integrated form an additional refinancing option (weakly) increases its value. Hence the Integrated form's joint profit maximization incentives are less important, which explains the lower critical cannibalization threshold γ^*. Finally, we have verified that the equilibrium maps for the other model parameters are directionally consistent with the ones discussed in Section 4. However, for brevity we do not show all of the results here (the equilibrium maps for the remaining parameters are available upon request).

[10] Another natural alternative would be to allow both the integrated firm and the large, separated firm to recapitalize at the time of option exercise with respect to their full collateral pool at that time, i.e., $1 + \pi$ for the integrated firm and 1 for the large, separated firm. However, this would be significantly less tractable and is not expected to affect the qualitative results since, like the alternative analyzed here, it maintains comparability across the organizational forms in overall capital structure flexibility.

6 Conclusion

This paper provides a first step toward analyzing how capital structure and organizational design jointly affect the value of the innovation process. We consider an Integrated form, in which all activities are operated in a single firm, and a Non-Integrated form, in which the innovative activity is instead operated by a small, stand-alone firm. For each organizational form, there are three sources of value: the value of assets in place, the value of the innovation option, and the value of net tax benefits. Non-Integration removes overhang from the exercise decision, maximizing pure option value and creating more capital structure flexibility. On the other hand, Integration best protects assets in place by taking joint profit considerations into account. These forces drive different organizational equilibria depending on firm and product market characteristics.

The analysis provides a number of unique empirical predictions. Notably, we find starkly different risk implications. Higher cash flow risk favors Non-Integration, while higher obsolescence risk favors Integration. In addition, since Non-Integration best maximizes financial flexibility, an increase in net tax benefits (due to lower tax rates or higher bankruptcy costs) makes Non-Integration more likely. Moreover, we establish that alliances organized as licensing agreements or revenue sharing contracts can better balance the different sources of value, and thus may dominate more traditional forms of organization. Our results should prove useful for future empirical investigations of whether successful innovation occurs inside or outside existing incumbent firms across different types of markets, as well as investigations of the role and structuring of alliances.

While we capture important economic forces, which produce numerous, novel implications, we have left out many other frictions and imperfections. To focus on interactions of corporate financing and investment, we have not modeled managerial skills. Depending on compensation costs, this could provide the Non-Integrated form with an additional source of value since it can hire two managers with human capital that best fits the different corporate activities. Also, for tractability we have not examined activities with imperfectly correlated cash flows. This could provide the Integrated form with an additional source of value in that more negatively correlated cash flows create additional debt capacity. However, the net effect of this additional debt capacity could be ambiguous since the inclination to issue more debt should provoke more debt overhang but the exercise threshold itself should decline relative to the small, stand-alone firm because the option's payoff in the integrated firm increases when the correlation decreases. Finally, we note that, for the sake of fairness across forms, we have not recognized transactions costs of either integrating or separating activities because they would drive optimal organizational design in less subtle ways. Overall, these observations suggest that extensions along some of these dimensions will prove fruitful for future research.

Appendix A. Derivations of Lemma 1 and Proposition 1

Within the present model, the exponential law holds for all $t \geq 0$:

$$\mathrm{P}(Y_t = 0) \;=\; e^{-\rho\, t} \; . \tag{A.1}$$

This implies that at any time t the expected time to obsolescence of the innovation option is given by:

$$\mathrm{E}\,[\mathcal{T}_Y] \;=\; \int_t^{\infty} \rho\,(s-t)\, e^{-\rho\,(s-t)}\, ds \;=\; \frac{1}{\rho}\; . \tag{A.2}$$

Consistent with economic intuition, equation (A.2) indicates that the expected time to preemption is inversely related to the risk of preemption.

Recall that for $t < \mathcal{T}_G$ the small firm invests to maximize the (equity) value of the levered assets obtained from exercise as long as exercise takes place prior to obsolescence (i.e., for $\mathcal{T}_G < \mathcal{T}_Y$):

$$V_S(X) \;=\; \sup_{\mathcal{T}_G} \mathrm{E}\left[\, 1_{\mathcal{T}_G < \mathcal{T}_Y}\, e^{-r\mathcal{T}_G} \left(V_S^+(X_{\mathcal{T}_G}) - \,\kappa\right)\right] \; . \tag{A.3}$$

On the other hand, if $\mathcal{T}_G > \mathcal{T}_Y$, then the small firm vanishes: $V_S(X) \;=\; 0$.

For $t > \mathcal{T}_G$, the small firm operates assets in place and hence its owners receive capital gains of $\mathrm{E}\left[dV_S^+(X)\right]$ and cash flows $(1-\tau)\,\pi X \,+\, \tau\, C_S^+$ over each time interval dt. The required rate of return for investing in the small firm is the risk-free rate r. Thus, the Bellman equation in the continuation region (i.e., for $t < \mathcal{T}_D$) is:

$$r\, V_S^+(X)\, dt \;=\; \mathrm{E}\left[dV_S^+(X)\right] \,+\, \left[(1-\tau)\,\pi X \,+\, \tau\, C_S^+\right] dt\; . \tag{A.4}$$

Applying Ito's lemma to expand the right-hand side of the Bellman equation, it is straightforward that the value of the matured, small firm before default (i.e., for $X > \underline{X}_S^+$) satisfies:

$$r\, V_S^+(X) \;=\; (1-\tau)\,\pi\, X + \tau\, C_S^+ + \mu\, X\, \frac{\partial V_S^+(X)}{\partial X} + \frac{1}{2}\,\sigma^2 X^2\, \frac{\partial^2 V_S^+(X)}{\partial X^2}\; . \tag{A.5}$$

The ordinary differential equation has a solution of the form:

$$V_S^+(X) \;=\; \pi\,\Lambda\, X + \frac{\tau\, C_S^+}{r} + A_1\, X^{\vartheta'} \,+\, A_2\, X^{\xi'}\;, \tag{A.6}$$

where $\vartheta' < 0$ is given in (4) and $\xi' > 1$ is given by: $\xi' \;=\; (\frac{1}{2} - \mu/\sigma^2) \,+\, \sqrt{(\frac{1}{2} - \mu/\sigma^2)^2 + 2\,r/\sigma^2}\;.$ The constants A_1 and A_2 are determined by the value-matching conditions of the levered firm:

$$V_S^+(\underline{X}_S^+) \;=\; \pi\,(1-\alpha)\Lambda\, \underline{X}_S^+\;, \tag{A.7}$$

$$\lim_{X\to\infty} V_S^+(X) \;=\; \pi\,\Lambda\, X \,+\, \frac{\tau\, C_S^+}{r}\; . \tag{A.8}$$

The first condition captures the value of the firm net of bankruptcy costs that will be transferred to the new owners in case of default. The second condition states that firm value is bounded above by the default-risk-free value of assets and tax shields, which implies that $A_2 = 0$. Solving the first equation for the remaining unknown constant A_1 yields firm value after investment, given in the first equation of Lemma 1.

Similar arguments as in equations (A.3), (A.5), and (A.6) yield equity value after investment, given in the second equation of Lemma 1, if we solve the value-matching conditions for equity:

$$E_S^+(\underline{X}_S^+) \;=\; 0 \;, \tag{A.9}$$

$$\lim_{X\to\infty} E_S^+(X) \;=\; \pi\,\Lambda\,X \;-\; \frac{(1-\tau)\,C_S^+}{r} \;. \tag{A.10}$$

The first condition ensures that equity is worthless in case of default, while the second equation corresponds to a no-bubble condition implying again that $A_2 = 0$. In addition to these value-matching conditions, equity value satisfies an optimality condition. That is, the default threshold that maximizes equity value solves the smooth-pasting condition:

$$\left.\frac{\partial E_S^+(X)}{\partial X}\right|_{X=\underline{X}_S^+} \;=\; 0 \;, \tag{A.11}$$

which implies the closed-form solution for $\underline{X}_S^+$ given in Lemma 1. Substituting the result for $\underline{X}_S^+$ into $V_S^+(X)$, the first-order condition of firm value after investment with respect to C_S^+ is given by:

$$\frac{\partial V_S^+(X)}{\partial C_S^+} \;=\; 0 \;, \tag{A.12}$$

which can be solved analytically to produce the closed-form solution for C_S^+ given in Lemma 1. This solution is indeed optimal given that it is straightforward to verify that the second-order condition for this optimization problem is negative.

Plugging the expressions for C_S^+ and $\underline{X}_S^+$ into equation (2) and simplifying yields:

$$V_S^+(X) \;=\; X\left(1 + \frac{\tau}{1-\tau}\left(1 - \vartheta'\,(1-\alpha+\alpha/\tau)\right)^{1/\vartheta'}\right) \;, \tag{A.13}$$

which we can substitute into the solution for equation (A.3). In the continuation region of cash flow levels below which investment is optimal (i.e., $X < \overline{X}_S$), standard arguments imply that equity value satisfies the Bellman equation:

$$r\,V_S(X)\,dt \;=\; \mathrm{E}\left[dV_S(X)\right] \;. \tag{A.14}$$

Applying Ito's lemma to expand the right-hand side of the Bellman equation yields for $X < \overline{X}_S$:

$$r\,V_S(X) \;=\; \mu\,X\,\frac{\partial V_S(X)}{\partial X} \;+\; \frac{1}{2}\,\sigma^2 X^2\,\frac{\partial^2 V_S(X)}{\partial X^2} \;+\; \rho\left[0 - V_S(X)\right] \;, \tag{A.15}$$

which has a solution of the form:

$$V_S(X) \;=\; A_3\,X^{\vartheta} \;+\; A_4\,X^{\xi}\;, \tag{A.16}$$

where $\vartheta < 0$ and $\xi > 1$ are given in (32) and (15). The unknown constants A_3 and A_4 are determined by the value-matching conditions for the small firm's equity value:

$$\lim_{X\to 0} V_S(X) \;=\; 0\;, \tag{A.17}$$

$$V_S(\overline{X}_S) \;=\; V_S^{+}(\overline{X}_S) \,-\, \kappa\;. \tag{A.18}$$

The first condition stipulates that the option ought to be worthless when cash flows become arbitrarily small, implying that $A_3 = 0$. The second is a no-arbitrage condition as it says that, at the time of exercise, equity value before exercise, V_S, equals equity value after exercise, V_S^{+}, net of the exercise cost, κ. Solving the second equation for the remaining unknown constant, A_4, and substituting the optimal investment threshold $\overline{X}_S$ yields equity value before investment, given in the first equation of Proposition 1. In addition to the value-matching conditions, equity value satisfies an optimality condition. That is, the optimal exercise threshold solves the smooth-pasting condition:

$$\left.\frac{\partial V_S(X)}{\partial X}\right|_{X=\overline{X}_S} \;=\; \left.\frac{\partial V_S^{+}(X)}{\partial X}\right|_{X=\overline{X}_S}\;, \tag{A.19}$$

which implies the closed-form solution for $\overline{X}_S$ given in the last equation of Proposition 1. ∎

Appendix B. Derivations of Lemma 2, Lemma 3 and Proposition 2

Similar arguments as the ones used for deriving Lemma 1 can be used to derive the closed-form solutions given in Lemma 2 and Lemma 3. In particular, observe that if we replace the variable π, the size of the growth opportunity, in all expressions given in Lemma 1 by the variable $(1+\gamma)$, the reduced size of assets in place resulting from cannibalization due to option exercise, then we obtain all the expressions given in Lemma 2. Similarly, notice that we can replace the size of the growth opportunity, π, in all expressions given in Lemma 1 by the reduced size of assets in place resulting from cannibalization due to preemption, $(1+\delta)$, to produce closed-form solutions given in Lemma 3.

In the continuation region of cash flow levels below which investment is optimal (i.e., $X < \overline{X}_S$) for the small firm and above which default is optimal (i.e., $X > \underline{X}_L$) for the large firm, standard arguments imply that for all $t < \mathcal{T}_Y$ firm value satisfies:

$$r\,V_L(X)\,dt \;=\; \mathrm{E}\left[dV_L(X)\right] \;+\; \left[(1-\tau)\,X \;+\; \tau\,C_L\right]dt\;. \tag{B.1}$$

Ito's lemma then says that the ordinary differential equation before investment and preemption is:

$$r\,V_L(X) \;=\; (1-\tau)\,X \,+\, \tau C_L \,+\, \mu\,X\,\frac{\partial V_L(X)}{\partial X} \,+\, \frac{1}{2}\,\sigma^2 X^2\,\frac{\partial^2 V_L(X)}{\partial X^2} \,+\, \rho\,[V_L^\circ(X) - V_L(X)]\ . \tag{B.2}$$

It is easy to guess and verify that this ordinary differential equation has a solution of the form:

$$\begin{aligned} V_L(X) \;=&\; \Lambda\,X + \frac{\tau\,C_L}{r} + A_5\,X^{\vartheta} \,+\, A_6\,X^{\xi} \,+\, \frac{\rho}{r+\rho-\mu}\,\delta\,\Lambda\,X \\ &- \frac{\rho}{r+\rho-\vartheta'\mu-\frac{1}{2}\,\vartheta'\,(\vartheta'-1)\,\sigma^2}\left(\frac{\tau\,C_L}{r}+\alpha\,\Lambda\,\underline{X}_L^\circ\right)\left(\frac{X}{\underline{X}_L^\circ}\right)^{\vartheta'}\ . \end{aligned} \tag{B.3}$$

The first four terms on the right-hand side of (B.3) correspond to the ones in (A.6). In addition, the fifth and the sixth term capture preemption risk in that they reflect the cannibalization cost from the fringe and the change in net tax shields, respectively, which enter equation (B.2) via $V_L^\circ(X)$.

The unknown constants A_5 and A_6 are determined by the value-matching conditions:

$$V_L(\underline{X}_L) \;=\; (1-\alpha)\,\Lambda\,X_L + \frac{\rho}{r+\rho-\mu}\,\delta\,\Lambda\left[\underline{X}_L - \overline{X}_S\left(\frac{\underline{X}_L}{\overline{X}_S}\right)^{\xi}\right] + \gamma\,\Lambda\,\overline{X}_S\left(\frac{\underline{X}_L}{\overline{X}_S}\right)^{\xi}\ , \tag{B.4}$$

$$V_L(\overline{X}_S) \;=\; V_L^+(\overline{X}_S)\ . \tag{B.5}$$

The first condition captures the value of the firm net of bankruptcy costs that will be transferred to the new owners in case of default. In particular, it not only reflects the bankruptcy costs but also the two cannibalization costs induced by either the small firm's exercise or by the competitive fringe's preemption (i.e., the second and the third terms in equation (B.4)), which remain associated with the large firm's assets through and beyond the restructuring process. The second condition states that firm value is bounded above by its post-investment value given in Lemma 2, which excludes the growth option's payoff. Solving these two equations for the two unknown constants yields firm value before investment and preemption, given in the first equation of Proposition 2.

Similar arguments as in equations (B.1), (B.2), and (B.3) yield equity value before investment and preemption, given in Proposition 2, if we instead solve the value-matching conditions for equity:

$$E_L(\underline{X}_L) \;=\; 0\ , \tag{B.6}$$

$$E_L(\overline{X}_S) \;=\; E_L^+(\overline{X}_S)\ . \tag{B.7}$$

The first condition ensures that equity is worthless in case of default, while the second condition ensures that equity value is bounded above by its post-investment value given in Lemma 2. In addition to the value-matching conditions, equity value also needs to satisfy a smooth-pasting condition:

$$\left.\frac{\partial E_L(X)}{\partial X}\right|_{X=\underline{X}_L} \;=\; 0\ . \tag{B.8}$$

As equation (B.8) is non-linear in $\underline{X}_L$, we obtain a quasi-closed form solution in form of a fairly complex equation (available upon request), which renders numerical solution necessary. Therefore, the optimal coupon choice of the firm numerically maximizes $V_L(X)$ with respect to C_L at time zero:

$$\max_{C_L} V_L(X; C_L, \underline{X}_L, \underline{X}_L^+, \underline{X}_L^\circ, \cdot) \ , \tag{B.9}$$

which takes into account C_L's effect on the optimally chosen default boundaries $\underline{X}_L$, $\underline{X}_L^+$, and $\underline{X}_L^\circ$ for the various regions. The code for the numerical optimization is available upon request. ■

Appendix C. Derivations of Lemma 4, Lemma 5 and Proposition 3

Similar arguments as the ones used for deriving Lemma 1 can be used to derive the closed-form solutions given in Lemma 4. That is, if we replace the factor π, the size of the growth opportunity, in all expressions given in Lemma 1 by the factor $(1 + \pi + \gamma)$, the increased size of assets in place resulting from option exercise but net of cannibalization due to option exercise, then we obtain all the expressions given in Lemma 4. It turns out that the equations in Lemma 5 are actually identical to the ones in Lemma 3 up to some subscripts, namely, L and I, which accommodates different coupon choices and correspondingly different default thresholds. Apart from these differences, the large firm's value functions after preemption and the integrated firm's value functions after preemption ought to be the same since they did not invest and operate the same assets in place.

Similar arguments as in equations (B.1), (B.2), and (B.3) yield integrated firm value before investment and preemption, given in Proposition 3, if we solve the modified value-matching conditions:

$$V_I(\underline{X}_I) \ = \ (1-\alpha)\,\Lambda\, X_I + \frac{\rho}{r+\rho-\mu}\,\delta\,\Lambda\,\underline{X}_I \ , \tag{C.1}$$

$$V_I(\overline{X}_I) \ = \ V_I^+(\overline{X}_I) \ . \tag{C.2}$$

Equation (C.1) captures the value of the firm net of bankruptcy costs that will be transferred to the new owners in case of default. In particular, it only reflects the bankruptcy costs and the cannibalization cost induced by the competitive fringe's preemption, which remain associated with the large firm's assets through and beyond the restructuring process, because the innovation option cannot be brought through the restructuring process. An alternative assumption, which we have explored in unreported simulations, is to partially transfer the option to the new owners, which corresponds to simply including a fractional value of the last two terms in (B.4) in (C.1). However, all the results are qualitatively very similar to the ones reported in the paper and hence suppressed for brevity. The intuition is that transferring some of the option to the debtholders increases their recoveries, which are multiplied by a fairly small number (default probability) for computing time zero firm value.

The small increase in firm value relative to Non-Integration lowers the cost of debt (credit spread) for a given coupon, which, in equilibrium, increases the optimal coupon and hence magnifies the overhang problem (i.e., a decrease in firm value relative to Non-Integration). Equation (C.2) states that firm value is bounded above by its post-investment value given in Lemma 4 net of the capital outlay required for option exercise. Solving these two equations for the two unknown constants yields firm value before investment and preemption, given in the first equation of Proposition 3.

Similar arguments as in equations (B.1), (B.2), and (B.3) yield equity value before investment and preemption, given in Proposition 3, if we instead solve the value-matching conditions for equity:

$$E_I(\underline{X}_I) \;=\; 0 \;, \tag{C.3}$$

$$E_I(\overline{X}_I) \;=\; E_I^+(\overline{X}_I) \;-\; \kappa \;. \tag{C.4}$$

The first condition ensures that equity is worthless in case of default, while the second condition ensures that equity value pastes correctly to its post-investment value given in Lemma 4 net of the equity-financed exercise cost, κ. In addition to these two value-matching conditions, optimality requires that equity value satisfies smooth-pasting conditions for default and investment:

$$\left.\frac{\partial E_I(X)}{\partial X}\right|_{X=\underline{X}_I} \;=\; 0 \;, \tag{C.5}$$

$$\left.\frac{\partial E_I(X)}{\partial X}\right|_{X=\underline{X}_I} \;=\; \left.\frac{\partial E_I^+(X)}{\partial X}\right|_{X=\underline{X}_I} \;. \tag{C.6}$$

Equations (C.5) and (C.6) are non-linear in $\underline{X}_I$ and $\overline{X}_I$, respectively, so that we obtain only quasi-closed form solutions in form of two tedious equations (available upon request). Non-linearity renders numerical solution necessary. Therefore, the optimal coupon choice of the firm numerically maximizes $V_I(X)$ with respect to C_I at time zero:

$$\max_{C_I} V_I(X; C_I, \underline{X}_I, \underline{X}_I^+, \underline{X}_I^\circ, \overline{X}_I, \cdot) \;, \tag{C.7}$$

which is clearly similar to the problem in equation (B.9). Yet, an important difference is that the optimal choice of C_I incorporates not only its effect on the optimally chosen default boundaries $\underline{X}_I$, $\underline{X}_I^+$, and $\underline{X}_I^\circ$ for the various regions, but also its effect on the jointly optimal investment boundary $\overline{X}_I$, which uniquely delivers endogenous overhang effects of the Integrated form. The numerical procedure for the constrained optimization is suppressed here but available upon request. ∎

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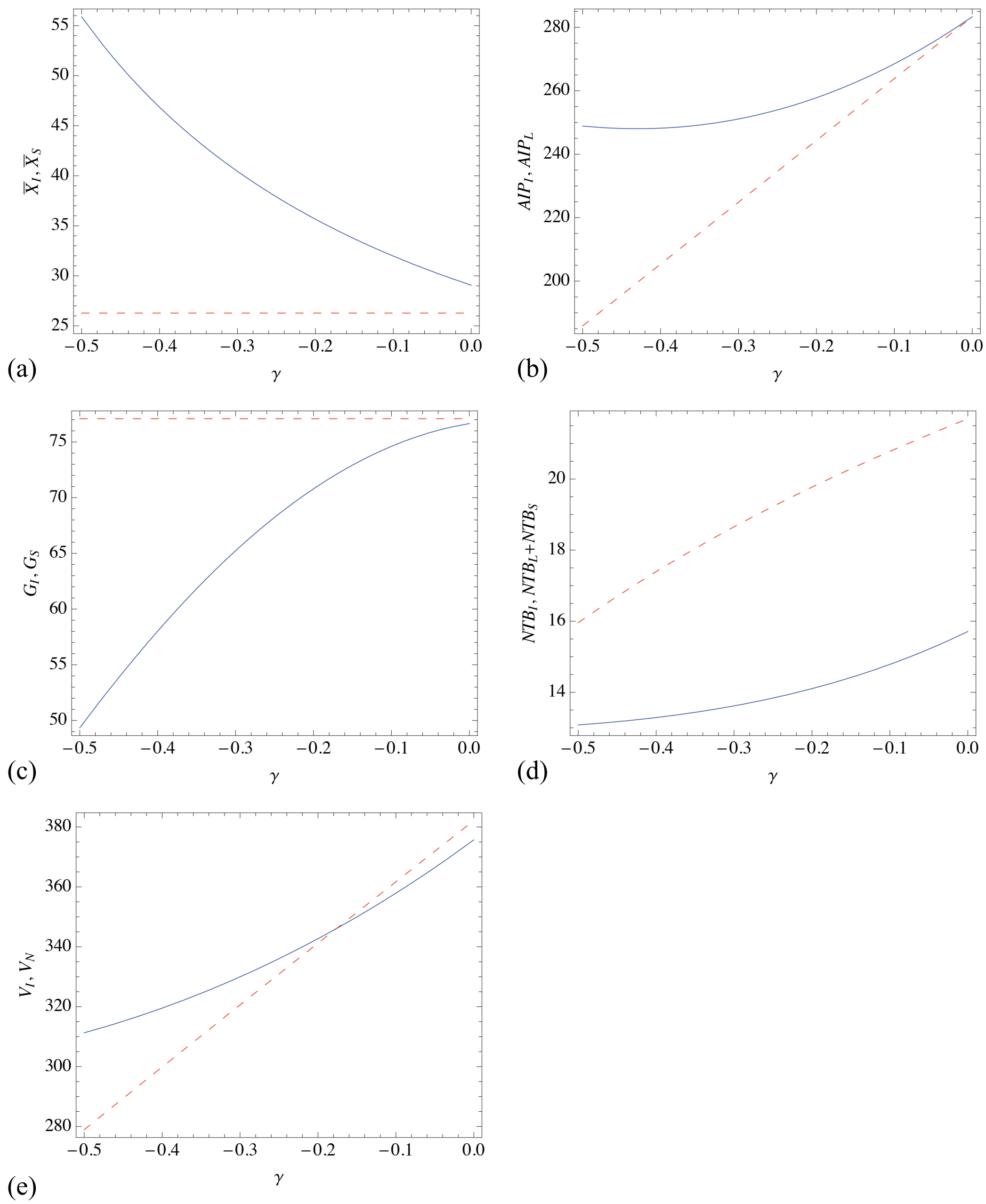

Fig. 1. Shows the effect of cannibalization intensity, γ, on optimal exercise times and values. All figures use a baseline environment in which cash flows start at $X = \$20$, the risk-free interest rate is $r = 7\%$, the growth rate of cash flows is $\mu = 1\%$, the volatility of cash flows is $\sigma = 30\%$, the risk of obsolescence is $\rho = 10\%$, the corporate tax rate is $\tau = 15\%$, the proportional cost of bankruptcy is $\alpha = 30\%$, the investment factor is $\pi = 100\%$, the investment cost is $\kappa = \$225$, and the cannibalization effect from obsolescence is $\delta = 0\%$.

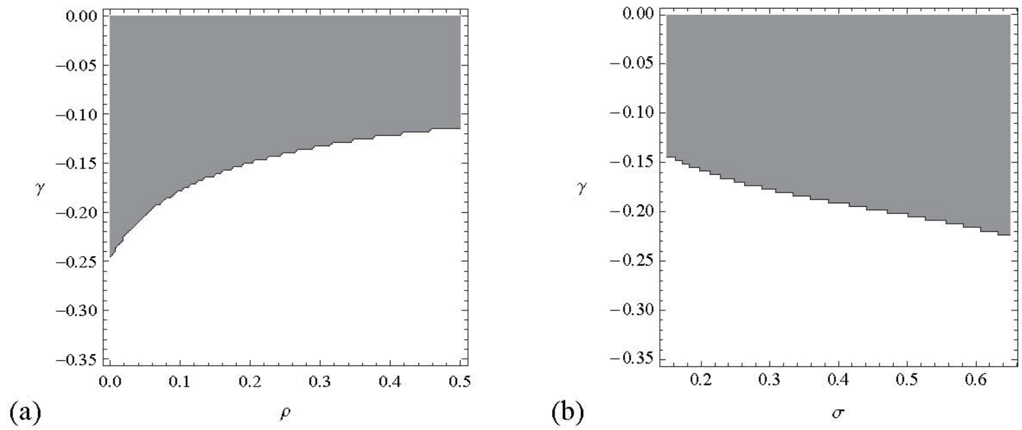

Fig. 2. Equilibrium maps showing the optimal organizational form as a function of two parameters holding all others constant. Non-Integration is optimal in shaded regions. Panel (a) varies cannibalization intensity, γ, and preemption risk, ρ. Panel (b) varies γ and cash flow uncertainty, σ. All parameter values are set as in the baseline environment.

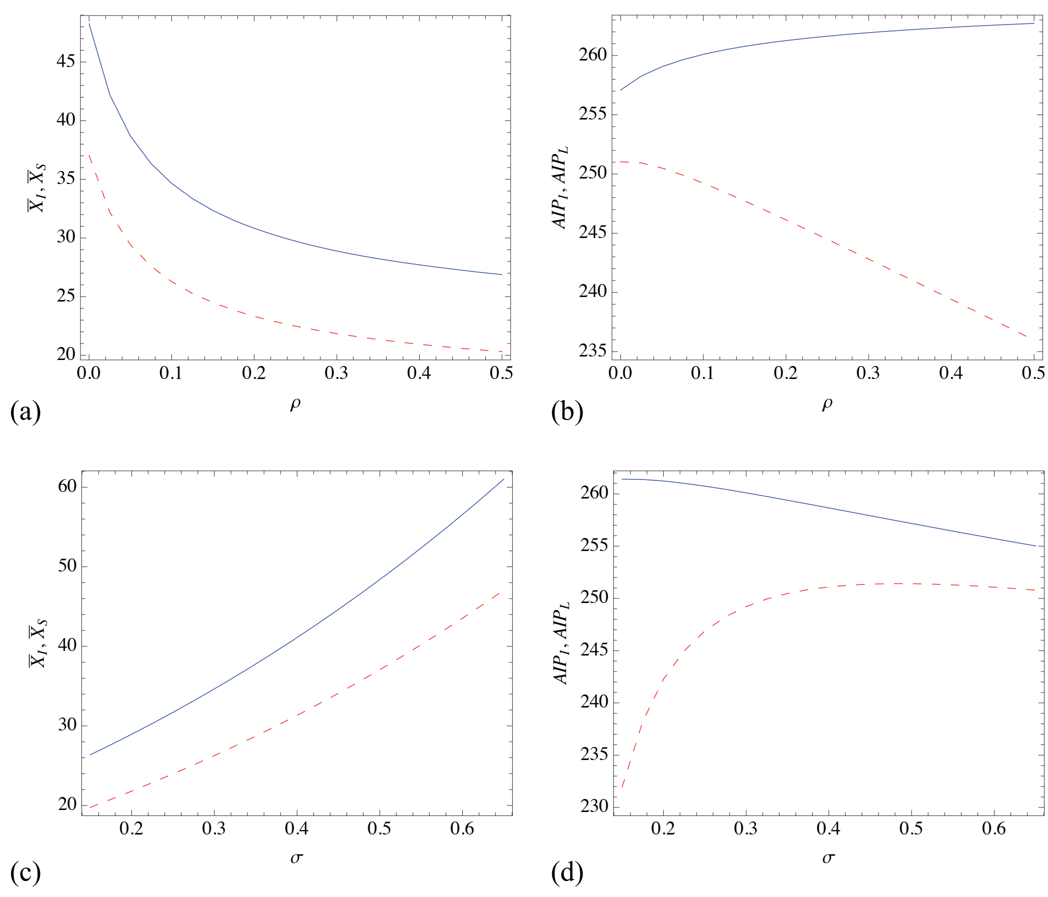

Fig. 3. Shows the effect of preemption risk, ρ, and cash flow uncertainty, σ, on optimal exercise times and assets in place values. All parameter values are set as in the baseline environment.

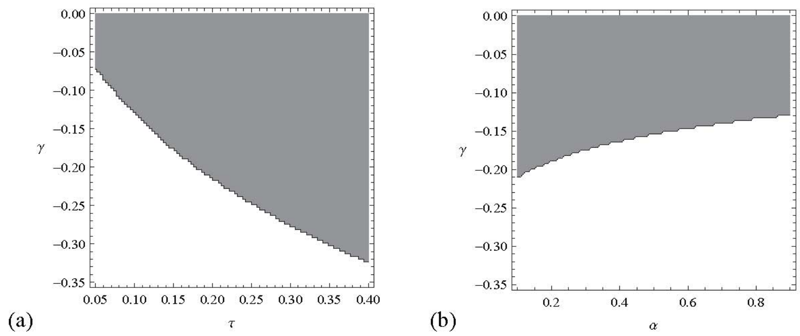

Fig. 4. Equilibrium maps showing the optimal organizational form as a function of two parameters holding all others constant. Non-Integration is optimal in shaded regions. Panel (a) varies cannibalization intensity, γ, and the corporate tax rate, τ. Panel (b) varies γ and bankruptcy costs, α. All parameter values are set as in the baseline environment.

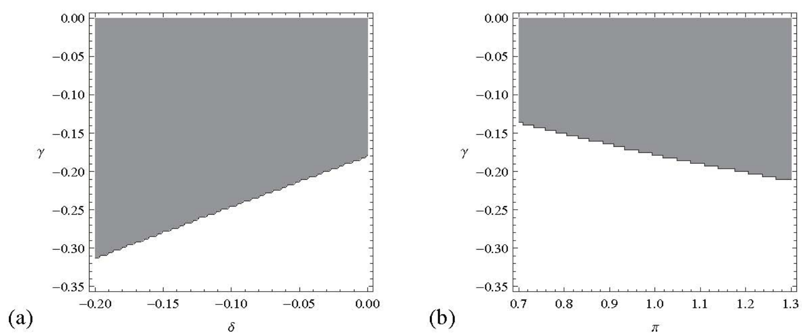

Fig. 5. Equilibrium maps showing the optimal organizational form as a function of two parameters holding all others constant. Non-Integration is optimal in shaded regions. Panel (a) varies cannibalization intensity, γ, and preemption cannibalization, δ. Panel (b) varies γ and the size of the growth option, π. All parameter values are set as in the baseline environment.

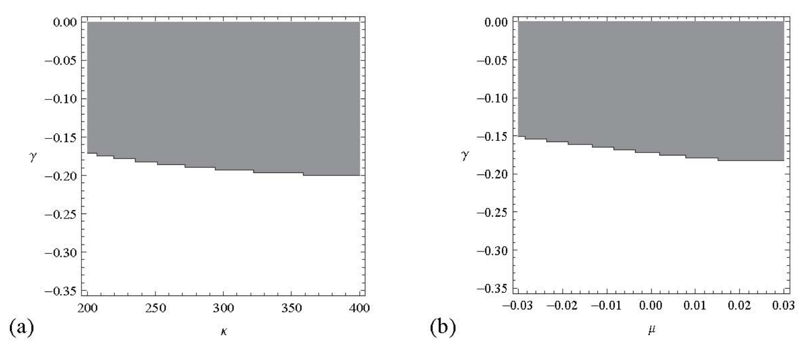

Fig. 6. Equilibrium maps showing the optimal organizational form as a function of two parameters holding all others constant. Non-Integration is optimal in shaded regions. Panel (a) varies cannibalization intensity, γ, and the cost of option exercise, κ. Panel (b) varies γ and the growth rate of cash flows, μ. All parameter values are set as in the baseline environment.

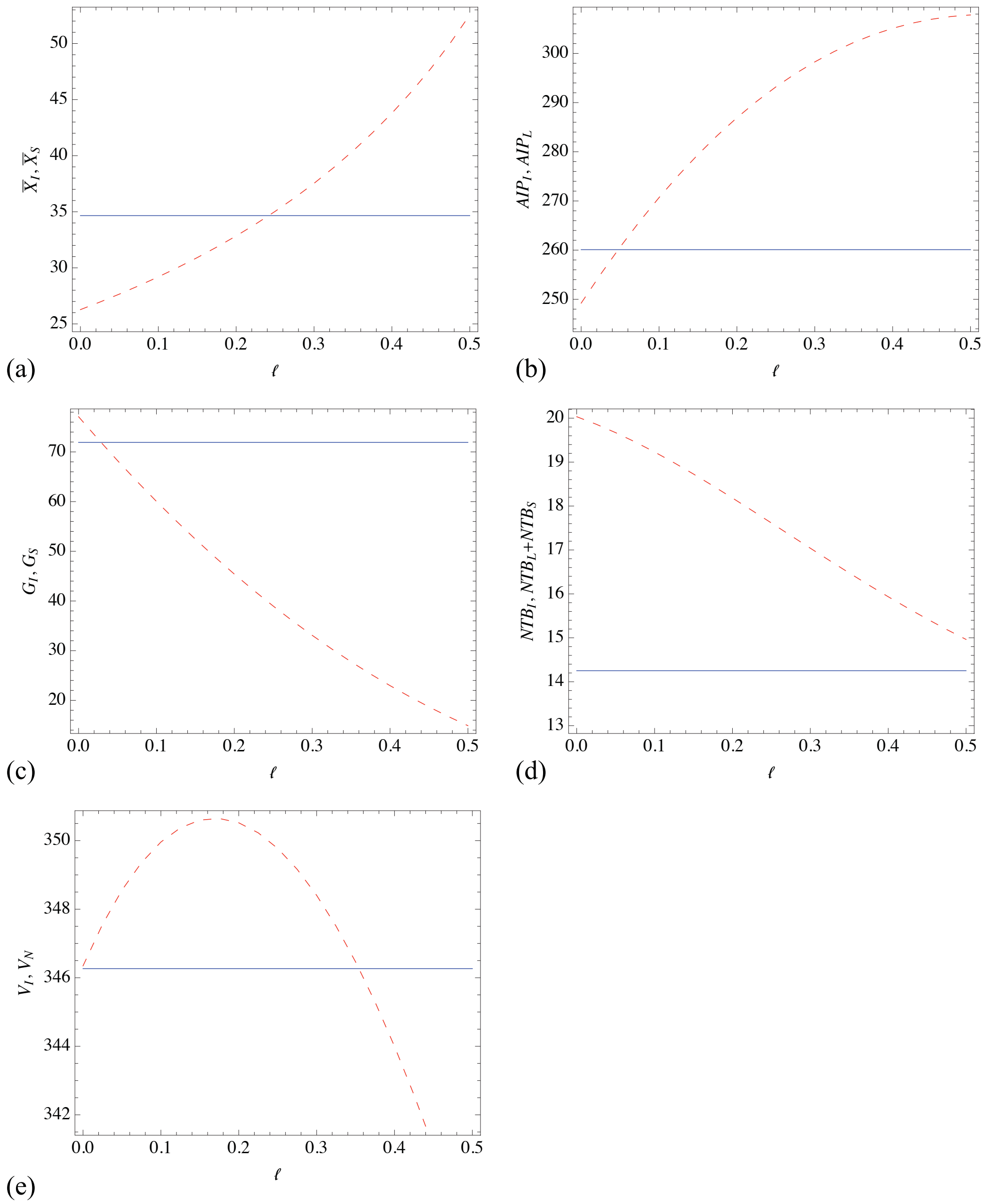

Fig. 7. Shows the effect of the licensing proportion, ℓ, on optimal exercise times and values. All parameter values are set as in the baseline environment.

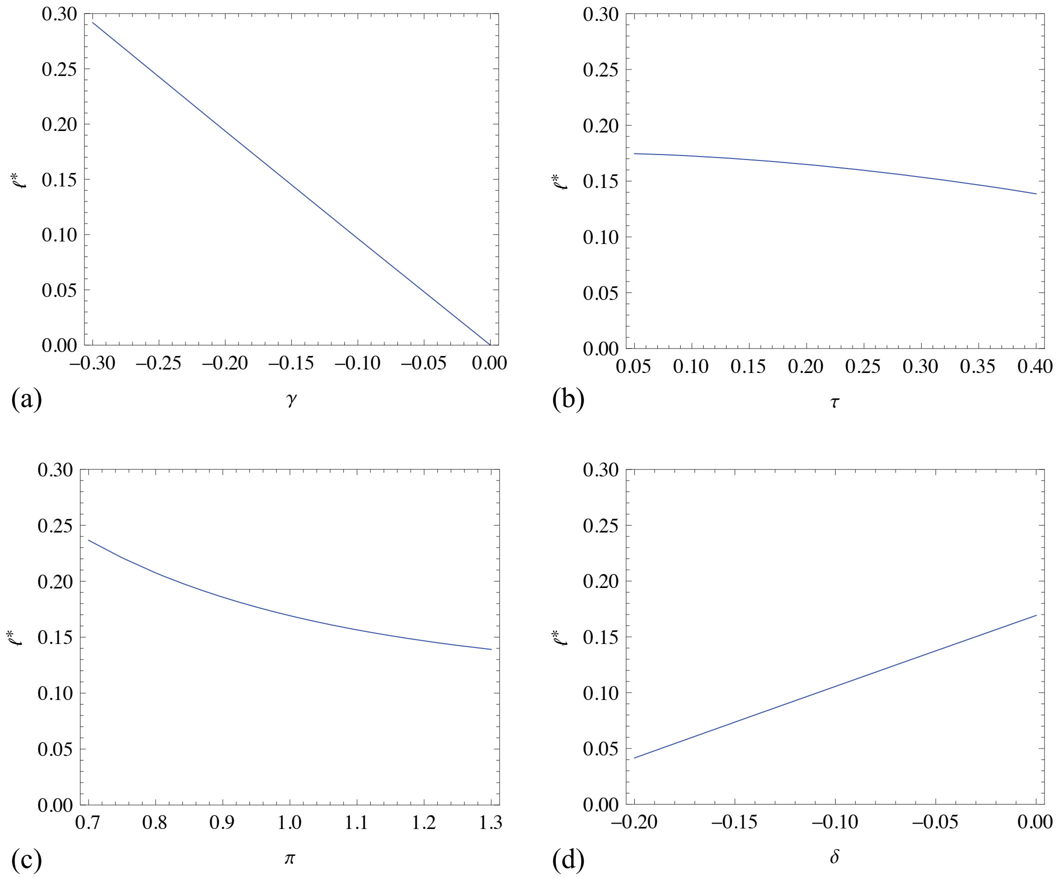

Fig. 8. Shows the effect of cannibalization intensity, γ, the corporate tax rate, τ, the size of the growth option, π, and preemption cannibalization, δ, on the optimal licensing proportion, ℓ^*. All parameter values are set as in the baseline environment.

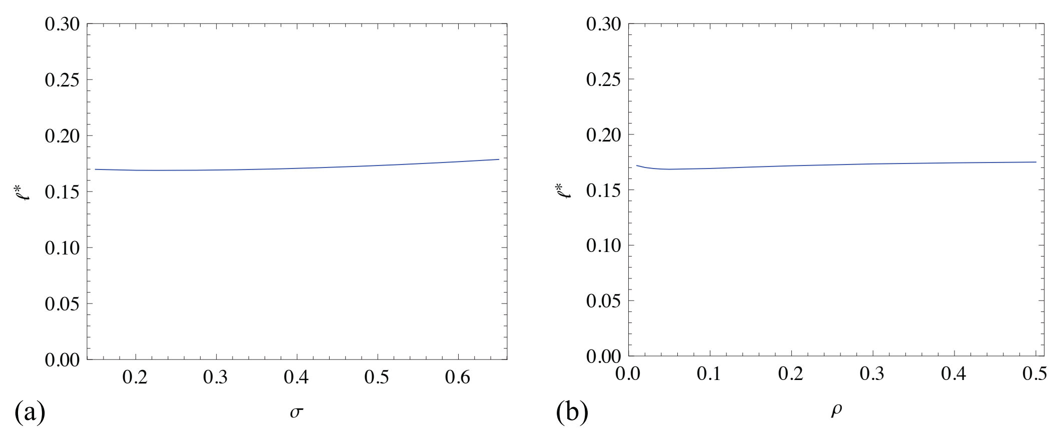

Fig. 9. Shows the effect of cash flow uncertainty, σ, and preemption risk, ρ, on the optimal licensing proportion, ℓ^*. All parameter values are set as in the baseline environment.

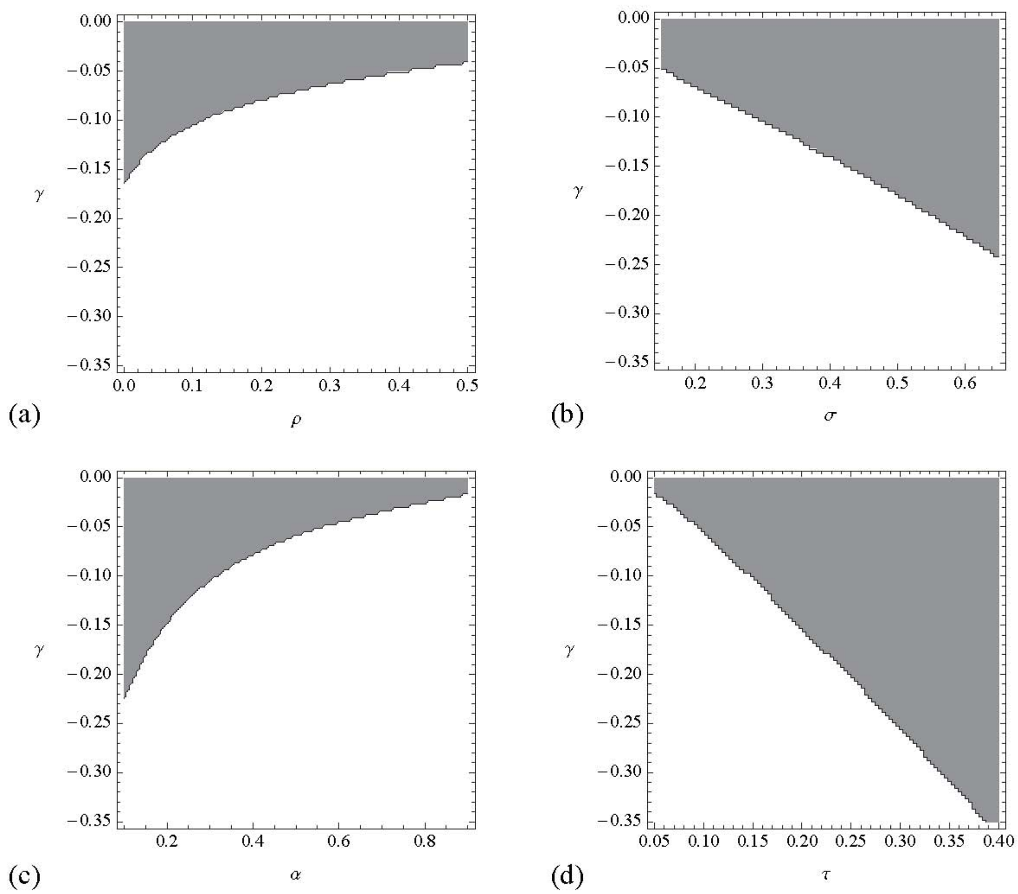

Fig. 10. Equilibrium maps showing the optimal organizational form as a function of two parameters holding all others constant for the model extension on Alternative Financing Arrangements. Non-Integration is optimal in shaded regions. Panel (a) varies cannibalization intensity, γ, and preemption risk, ρ. Panel (b) varies γ and cash flow uncertainty, σ. Panel (c) varies γ and bankruptcy costs, α. Panel (d) varies γ and the corporate tax rate, τ. All parameter values are set as in the baseline environment.